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SelfStudy Series

A2 – Fractions, Ratios & Percentages

version 1.04

Contents

PREFACE

Dear parents, guardians and teachers. Thank you for purchasing this study guide directly from algebrains.com. Our SelfStudy guides are available exclusively from algebrains.com (or from our offices) and have been priced to encourage greater accessibility from many students and their families who will benefit from our content. By purchasing directly, you are also contributing and supporting our mission in strengthening the delivery of Maths & Financial Education to children & young-adults in Britain (and throughout the world).

Our SelfStudy series have been written for students as a reference to teach them how to tackle mathematical challenges via step-by-step illustrations. Our materials have been designed to help parents to easily understand the workings too, to help you coach your child.

We have kept the content as concise and as pictorial as possible…so that our examples are easy to follow…therefore easy to understand and apply! We have also decided not to distract the students with elaborate colours as their exam papers will be in black & white.

Should you choose to complement your child's study with our classroom or webinar sessions, your child will also have access to additional illustrated workings for all questions that we shall practise.

Regardless of your child's level, whether a beginner or advanced…we firmly believe that our learning materials coupled with frequent practise will transform your young ones into numerically competent magicians!

Good luck and enjoy learning!

Ying & Jerry

What are Fractions?

Let's imagine a cake to be shared between friends;

- if there are two persons to share, then each would get half or $\frac{1}{2}$
- if there are four persons to share between them, each would get a quarter or $\frac{1}{4}$

This means the greater the number of friends to share, the smaller the portion each shall receive. This is the basis of Fractions….it is essentially a Division!

We must become comfortable with the way fractions are presented. Using the above as examples, the top value in the fraction (the 1's) is called the "NUMERATOR". The bottom value (the 2 and 4) is called the "DENOMINATOR"

$$\frac{\textbf{Numerator}}{\textbf{Denominator}} = \text{Numerator} \div \text{Denominator} = \text{Numerator} \setminus \text{Denominator}$$

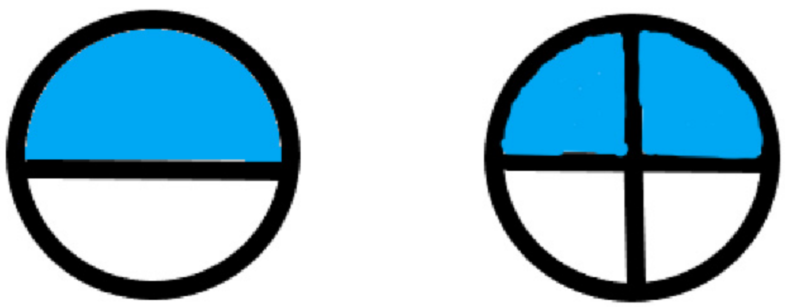
Notice from the above expressions, that fractions are essentially equivalent to Divisions. In fact, you should perceive fractions, ratios and percentages as different ways to express a Division! …simple!

Equivalent Fractions:

Equivalent is defined as 'same' or 'equal'….so Equivalent Fractions may appear different from each other, but they represent the same / equivalent value.

To demonstrate equivalency…study the picture below of two cakes being divided up.

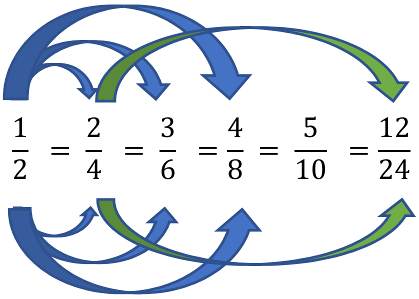
- Cake on left has been sliced into two-halves. You'll get the blue slice, which is $\frac{1}{2}$
- Cake on the right has been sliced into four-quarters. If you get two-blue slices, this is expressed as $\frac{2}{4}$ … but this is in fact the same quantity as $\frac{1}{2}$ of a cake



In otherwords, the fractions $\frac{2}{4}$ and $\frac{1}{2}$ are of the same value, they are equivalent!

The ONLY way to produce an Equivalent Fraction is to MULTIPLY or DIVIDE the numerator AND denominator by the SAME number. (Not to add or subtract the same number from numerator and denominator).

For example:



$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{12}{24}$$

Look at top smallest arrow where the numerator is multiplied by x2; we must do the same to the denominator too (therefore $\frac{1}{2}$ becomes $\frac{2}{4}$).

Similarly, if we multiply the numerator by x3, the denominator must also be multiplied by x3.

Conversely, we may work backwards by dividing the numerator and denominator of a fraction by the same number. This is called 'simplifying' a fraction. It makes the fraction simpler to handle…without changing its overall value!

Fundamentally, multiply and dividing the numerator and denominator of a fraction by the same number, does not change the fraction's value, it merely changes the way a fraction looks.

All six of the above fractions are the same value! Expressed in decimals, they are all equal to 0.5

In the following chapters we are going to learn how to manipulate fractions. Adding them, subtracting them, multiplying and dividing them.

You will soon begin to realise that fractions are not difficult and can be fun!

Fractions - Addition and Subtractions:

Before we can add or subtract fractions from each other…we must first prepare each fraction so they all have the SAME DENOMINATOR.

Example: $\frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{4}}$

Steps:

(1) Prepare the fractions so they share the same denominator values before addition.

Without changing the value of faction $\frac{1}{2}$ … we can multiply the numerator and denominator of this fraction by x2, transforming it to the equivalent of $\frac{2}{4}$:

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4}$$

(2) Once the denominators are aligned…they share the same value (4 in this case) …one can simply ADD the NUMERATORS of the fractions together. NEVER add the denominators…the denominator remains unchanged:

$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{2+1}{4} = \frac{\mathbf{3}}{\mathbf{4}}$$

Well done for showing your workings to arrive at the final answer of $\frac{3}{4}$

Example: $\frac{\mathbf{5}}{\mathbf{7}} + \frac{\mathbf{8}}{\mathbf{15}}$

Steps:

(1) Prepare the above fractions so they share the same common denominator. One way of achieving this quickly is to multiply the numerator and denominator of each fraction by the denominator of the other.

Thus $\frac{5}{7}$ can be converted into the equivalent of $\frac{75}{105}$ by multiply the numerator and denominator by x15:

$$\frac{5}{7} = \frac{5 \times 15}{7 \times 15} = \frac{75}{105}$$

Similarly, $\frac{8}{15}$ can be converted into the equivalent of $\frac{56}{105}$ by multiply the numerator and denominator by x7:

$$\frac{8}{15} = \frac{8 \times 7}{15 \times 7} = \frac{56}{105}$$

(2) Once the denominators share the same value (105 in this case)…one can simply add ONLY the Numerators of the fractions together:

$$\frac{5}{7} + \frac{8}{15} = \frac{75}{105} + \frac{56}{105} = \frac{75 + 56}{105} = \mathbf{\frac{131}{105}}$$

The answer is $\frac{131}{105}$. However, we can further simplify this Improper fraction given $\frac{131}{105}$ is greater than ONE. A fraction where its magnitude is greater than 1 is called an Improper fraction. Conversely, fractions where its magnitude is between 0 to 1 are called Proper fractions.

Remember $\frac{131}{105}$ is really a division, 131 ÷ 105. If you apply your long-division technique, only a 'single' multiple of 105 can be squeezed into 131, leaving a remainder of 26. We can thus simplify $\frac{131}{105}$ into the following notation:

$$\frac{131}{105} = \mathbf{1\frac{26}{105}}$$

This notation is equivalent to 1 + $\frac{26}{105}$. We have merely extracted-out an integer whole number and expressed the rest as a proper fraction.

Let's do a subtraction:

Example: $\mathbf{\frac{5}{7} - \frac{8}{15}}$

(1) As from the prior example, we know the equivalent fractions are $\frac{75}{105}$ and $\frac{56}{105}$

(2) Now subtract the numerators …simple!

$$\frac{5}{7} - \frac{8}{15} = \frac{75}{105} - \frac{56}{105} = \frac{75 - 56}{105} = \mathbf{\frac{19}{105}}$$

Well done for showing your workings to arrive at the final answer of $\frac{19}{105}$

Fractions - Multiplication and Division:

Multiply and dividing fractions is relatively simple and may firstly require you to 'simplify' fractions into a simpler equivalent, before the actual multiplication or division is performed.

Example: $\frac{3}{21} \times \frac{7}{5}$

Steps:

(1) Identify whether any of the fractions in the question can be further simplified to a 'simpler' equivalent. In this example, we can see that $\frac{3}{21}$ can be simplified into the equivalent of $\frac{1}{7}$ (by dividing numerator and denominator by ÷3).

$$\frac{3}{21} \times \frac{7}{5} = \frac{1}{7} \times \frac{7}{5}$$

(2) Multiplying fractions are easier (than adding or subtracting fractions) because you do NOT need to prepare the denominators to become the same value. You simply multiply numerator vs numerator, and denominator vs denominator:

$$\frac{3}{21} \times \frac{7}{5} = \frac{1}{7} \times \frac{7}{5} = \frac{1 \times 7}{7 \times 5}$$

(3) Those of you with a sharp eye may have noticed that in the above right most expression the numerator and denominator both contain x7 multiples. One can simplify this expression by dividing numerator and denominator simultaneously by ÷7…thus removing the 7's:

$$\frac{3}{21} \times \frac{7}{5} = \frac{1}{7} \times \frac{7}{5} = \frac{1 \times 7}{7 \times 5} = \frac{1}{5}$$

The answer is $\frac{1}{5}$

To divide one fraction over another, we must…

a) first invert (flip-over the numbers of) the second fraction (the one written after the ÷ sign) and then…
b) multiply the fractions using the technique illustrated earlier:

Example: $\frac{\mathbf{3}}{\mathbf{21}} \div \frac{\mathbf{7}}{\mathbf{5}}$

Steps

(1) Flip the second fraction upside-down and change its sign from ÷ to x

$$\frac{3}{21} \div \frac{7}{5} = \frac{3}{21} \times \frac{5}{7}$$

(2) Now simplify, multiply and further simplify the fractions:

$$\frac{3}{21} \div \frac{7}{5} = \frac{3}{21} \times \frac{5}{7} = \frac{1}{7} \times \frac{5}{7} = \frac{\mathbf{5}}{\mathbf{49}}$$

The answer is $\frac{5}{49}$

Example: $\mathbf{10} \div \frac{\mathbf{4}}{\mathbf{8}}$

Steps

(1) 10 can be expressed as a fraction $\frac{10}{1}$ (because 10 divided by 1 = 10). Flip the second fraction upside down and change its sign from ÷ to x

$$\frac{10}{1} \div \frac{4}{8} = \frac{10}{1} \times \frac{8}{4}$$

(2) Now simplify and multiply the fractions:

$$\frac{10}{1} \div \frac{4}{8} = \frac{10}{1} \times \frac{8}{4} = \frac{10}{1} \times \frac{2}{1} = \frac{\mathbf{20}}{\mathbf{1}} = \mathbf{20}$$

The answer is $\frac{20}{1}$ which is 20.

Ratios:

Ratios are frequently used to show proportionality or scale. For example, a cookery book instructs the mixing of margarine and flour (based on volume) at a ratio of **1:3** (pronounced one-to-three). This means for every cup of margarine we will mix 3 cups of flour.

There are several ways of representing ratios, generally we use "x:y". Other times we may use "x to y".

Sometimes ratios can be simplified, for example, the ratio 25:50 is the same as 1:2. This appears similar to simplifying fractions isn't it?. The best way to look at ratios is to treat them like fractions (a form of division). Thus x:y = $\frac{x}{y}$

Example:

Simplify each of the following ratios:

(a) 30:100 (b) 15:21 (c) 38:76

Steps:

(a) Like simplifying fractions, we need to determine the common factor for both terms (in this case 30 and 100 are both multiples of 10)
So simplify the ratio by dividing both terms by 10
30:100 = **3:10**

(b) Both 15 and 21 are multiples of 3, so we would start by dividing each term by 3
15:21 = **5:7**

(c) Both 38 and 76 are multiples of 2, therefore divide each term by 2
38:76 = 19:38
At this stage we can no longer simplify the ratio further by dividing each side by 2 as 19 is both an odd number and prime number. However, always check whether one side is a multiple of the other. Indeed, 38 is two-times multiple of 19 …we can therefore further simplify this to 1:2
38:76 = 19:38 = **1:2**

Example:

In a box of chocolates there are 20 strawberry flavoured ones, 10 lemon flavoured and 10 hazelnut flavoured chocolates. Find the ratio of:

(a) Strawberry to lemon flavour
(b) Lemon to whole-box of chocolates.

Steps:

(1) The ratio of strawberry to lemon is 20 strawberry to 10 lemons (20:10) or can be simplified to 2 to 1 (**2:1**).

(2) The ratio of Lemon to whole box of chocolates will therefore be 10 Lemons to 40 chocolates in the box (10:40) or can be simplified to **1:4**.

Example:

Amy loves mixing fruit cocktails. Her favourite drink is mixing orange juice and pineapple juice in a ratio of 3:7. Determine the volume of pineapple juice required with 600ml of orange juice?

Steps

(1) Express the ratio as a fraction. include units of measurement:

$$\frac{3\ parts\ orange}{7\ parts\ pineapple}$$

(2) Express the question next to this as an Equivalent Fraction (you have been asked to derive the volume of pineapple juice, so we shall denote this as '? ml'. Remember, whether you have 600ml of orange juice or 500litres of orange juice, you must still mix pineapple juice at a ratio (or fraction) of 3:7!

$$\frac{3\ parts\ orange}{7\ parts\ pineapple} = \frac{600ml\ orange}{?\ ml\ pineapple}$$

The above set-up is very common in fractions or ratios. You have essentially set-up a grid with four quadrants. If you know the values and 'units of measurement' for three grids, then you have enough information to determine the remaining grid:

A	C
B	D

C ÷ A x B = **D**

B ÷ A x C = **D**

OR

A	C
B	D

A ÷ C x D = **B**

D ÷ C x A = **B**

Can you see, that you can either go clockwise or anticlockwise from the empty cell D or B….you will arrive at the same answer. You will appreciate this more when you study Algebra and how to manipulate algebra or formulas in equations in the next guide A3- Algebra & Equations.

So applying this to the below Equivalent Fractions:

$$\frac{3\ parts\ orange}{7\ parts\ pineapple} = \frac{600ml\ orange}{?\ ml\ pineapple}$$

600ml ~~orange~~ ÷ 3 parts ~~orange~~ x 7 parts pineapple = **1400ml pineapple**

Keep the units-of-measurement in your workings as this will help you determine the unit-of-measurement for your answer! Sense heck your answer before finishing…is 600ml/1400ml equivalent to the fraction 3/7? Your answer should be ☺

Ratios are simply a form of fractions…and in questions where you are asked to apply a ratio to a measurement X to determine another Y…remember to use your skills of equivalence.

Percentages:

Percentage means "per a hundred" using % as the symbol to indicate that there is a hidden denominator of 100. So if you see the % sign next to a number; divided that number by 100 to convert your number into a decimal format…easy!

Rule to remember: 100% = $\frac{100}{100}$ = 1

similarly 20% = $\frac{20}{100}$ = 0.2

Example:

Convert 0.04 to percentage

Steps:

0.04 = 4 ÷ 100 = $\frac{4}{100}$ therefore = **4%**

Example:

Convert $\frac{1}{4}$ to percentage

Steps:

Using equivalent fraction, we can make the denominator from 4 to 100.

$\frac{1}{4} = \frac{25}{100}$ = **25%**

Example:

Becky bought a vase for £70.00. Three years later the value had increased in total by 25%. What is the new value of the vase?

Steps:

(1) The question is explicit that there was an increase in value, therefore we must seek an answer greater than the original starting value of £70.00.

(2) We need to find 25% of £70. This is the same as £70 $\times \frac{25}{100}$ or simplified as £70 $\times \frac{1}{4}$ = $£\frac{70}{4} = £\frac{35}{2} = £17.50$

So the price *increase* is £17.50.

Add the price increase to original price of £70.00, we then get the new value of **£87.50** (£70.00 + £17.50).

Example:

In a January Sale, Next (a retailer) is reducing the price of outdoor coats by 30%. The cost of a coat before the sale was £150.00. What is the value of the coat during the sale?

Steps:

(1) The question had clearly stated there is a reduction in price, hence we must ensure the final answer will be less than original cost of £150.00.

(2) We need to find 30% of £150.00.

$$£150 \times \frac{30}{100} = £150 \times \frac{3}{10} = £15 \times \frac{3}{1} = \frac{45}{1} = £45$$

(3) Reduction from original price means take away from value before sale:

£150 – £45 = **£105**

APPENDIX - formulas

To consolidate your knowledge, you must practise, practise and practise! Enquire about our classroom & webinar courses or Question & Answer materials…visit us at www.algebrains.com

You may have noticed some patterns emerging through our worked examples. Below are some expressions in algebraic form of how to do fractional additions, subtractions, multiplications and divisions. See if you can follow the formulas below …get familiar with them!

Addition formula:

$$\frac{a}{b} + \frac{c}{d} = \frac{(a \times d) + (c \times b)}{b \times d}$$

Subtraction formula:

$$\frac{a}{b} - \frac{c}{d} = \frac{(a \times d) - (c \times b)}{b \times d}$$

Multiplication formula:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Division formula:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$$

Changing the display of improper fractions:

$$a\frac{b}{c} = \frac{(a \times c) + b}{c}$$

Practice Questions:

Calculate & show workings…

Question 1:

Work out

(a) $\frac{4}{7}+\frac{4}{11}$
(b) $\frac{4}{7}-\frac{4}{11}$
(c) $\frac{4}{7}\times\frac{4}{11}$
(d) $\frac{4}{7}\div\frac{4}{11}$

Question 2:

In a classroom of 30 students, 14 are boys are 16 are girls. 10 of the girls like to play netball, 12 of the boys opt for football and the remaining students prefer to read. Please determine:

(a) the ratio of boys and girls
(b) What fraction of girls like playing netball?
(c) What percentage of the students like playing football.

Question 3:

There are 24 children in Anna's class. Three Eighths of the class have school lunch. How many students don't have school lunch?

Question 4:

Determine the missing value:

(a) 11% of 2400
(b) ____ % of 136 = 17
(c) 25% of ____ = 250

Question 5:

In a perfect pancake mixture, we mix 150g of flour with 50 ml of milk. Amy only has 30 ml of milk in her refrigerator. How much flour does she need to make pancakes?

Answers & Model Workings:

Question 1: Work out (a) $\frac{4}{7}+\frac{4}{11}$ (b) $\frac{4}{7}-\frac{4}{11}$ (c) $\frac{4}{7}\times\frac{4}{11}$ (d) $\frac{4}{7}\div\frac{4}{11}$	(a) We need to make the denominators of all terms to be the same when adding or subtracting fractions. $\frac{4}{7}+\frac{4}{11}=\frac{44}{77}+\frac{28}{77}=\mathbf{\frac{72}{77}}$ (b) $\frac{4}{7}-\frac{4}{11}=\frac{44}{77}-\frac{28}{77}=\mathbf{\frac{16}{77}}$ (c) When multiplying fractions we simply multiply numerator vs numerator, denominator vs denominator: $\frac{4}{7}\times\frac{4}{11}=\mathbf{\frac{16}{77}}$ (d) See how we have simplified the answer further more into an Improper fraction: $\frac{4}{7}\div\frac{4}{11}=\frac{4}{7}\times\frac{11}{4}=\frac{1}{7}\times\frac{11}{1}=\frac{11}{7}=\mathbf{1\frac{4}{7}}$
Question 2: In a classroom of 30 students, 14 are boys are 16 are girls. 10 of the girls like to play netball, 12 of the boys opt for football and the remaining students prefer to read. Please determine: (a) the ratio of boys and girls (b) What fraction of girls like playing netball? (c) What percentage of the students like playing football.	(a) boys vs girls: Therefore, 14:16 = **7:8** once simplified. (b) 10 girls like netball out of total of 16 girls, therefore expressed as a fraction $\frac{10}{16}=\frac{\mathbf{5}}{\mathbf{8}}$ once simplified (c) 12 boys likes football, therefore 12 out of 30 students like football. Express this as a fraction and then convert to percentage: $\frac{12}{30}=\frac{6}{15}=\frac{2}{5}=\frac{40}{100}=40\%$
Question 3: There are 24 children in Anna's class. Three Eighths of the class have school lunch. How many students don't have school lunch?	We have to work out number of children having school lunch first. $24\times\frac{3}{8}=9$ There are 9 children having school lunch, therefore the number of students not having school lunch would be 24 – 9 = **15**
Question 4: Work out the missing value: (a) 11% of 2400 (b) _____ % of 136 = 17 (c) 25% of _____ = 250	(a) Several ways of working this out, you could use decimal of 0.11 or fraction $\frac{11}{100}$ to represent 11%. I am going to use

	$\frac{11}{100} \times 2400 = \frac{11}{1} \times 24 = \mathbf{264}$ (b) As we know % can be converted between decimal and fractions and vice versa. How this question can be viewed is like an algebra equation, $\frac{x}{100}$ × 136 = 17 So we can multiply right hand side by 100 and then divide by 136 to get x. X = 100 × 17 ÷ 136 = 12.5 therefore x% = **12.5%** (c) Express your question as follows and determine y: $\frac{25}{100}$ × y = 250 to find y we can multiply right-hand-side by 100 and then divide by 25. y = 250 × 100 ÷ 25 = **1000**
Question 5: In a perfect pancake mixture, we mix 150g of flour with 50 ml of milk. Amy only has 30 ml of milk in her refrigerator. How much flour does she need to make pancakes?	Put this in a ratio format: Flour : Milk 150g : 50 ml ? : 30 ml To work out '?', we use $\frac{30ml}{50ml}$×150g = **90g** of flour needed.

Top Tips: Always remember to include your units-of-measurement in your workings and answers. Secondly, check your answer for reasonableness by inserting it back into the original questions….does it still work? Does it make sense?