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SelfStudy Series

A5 – **Shapes & Geometry**

version 1.04

Contents

PREFACE

Dear parents, guardians and teachers. Thank you for purchasing this study guide directly from algebrains.com. Our SelfStudy guides are available exclusively from algebrains.com (or from our offices) and have been priced to encourage greater accessibility from many students and their families who will benefit from our content. By purchasing directly, you are also contributing and supporting our mission in strengthening the delivery of Maths & Financial Education to children & young-adults in Britain (and throughout the world).

Our SelfStudy series have been written for students as a reference to teach them how to tackle mathematical challenges via step-by-step illustrations. Our materials have been designed to help parents to easily understand the workings too, to help you coach your child.

We have kept the content as concise and as pictorial as possible…so that our examples are easy to follow…therefore easy to understand and apply! We have also decided not to distract the students with elaborate colours as their exam papers will be in black & white.

Should you choose to complement your child's study with our classroom or webinar sessions, your child will also have access to additional illustrated workings for all questions that we shall practise.

Regardless of your child's level, whether a beginner or advanced…we firmly believe that our learning materials coupled with frequent practise will transform your young ones into numerically competent magicians!

Good luck and enjoy learning!

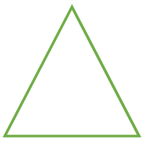
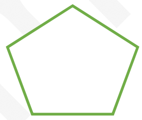
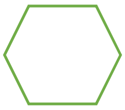
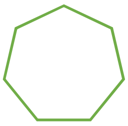
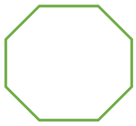
Ying & Jerry

Shapes:

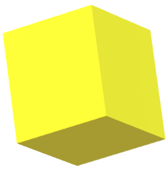
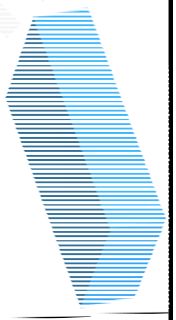
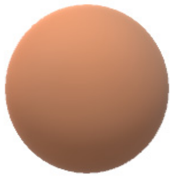
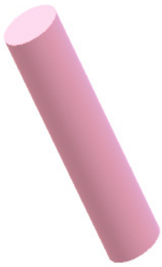
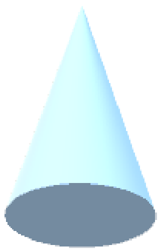
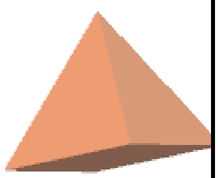
Two-dimensional (2D) shapes are flat shapes. For example, a football pitch viewed from the sky is only a rectangular 2D shape.

Three-dimensional (3D) shapes are solids. For example, traffic cones, spherical oranges, blocks of bricks or quadrants of cheese.

Below are the names of some 2D polygons

Triangle 3 sides 	**Quad**rilateral 4 sides 	**Pent**agon 5 sides 
Hexagon 6 sides 	**Hept**agon 7 sides 	**Oct**agon 8 sides 

Below are the names of some 3D polygons

Cube A box, all edges have equal lengths. Each face is a square. 	**Cuboid** Rectangular solid. A cross-section is also rectangular. 	**Sphere** A ball, a circular solid. 
Cylinder A tube, having a uniform circular cross-section. 	**Cone** Has a circular base and tapers to a point (apex) at the top. 	**Pyramid** A flat base and tapered to a point (apex) at the top. 

Questions on 2D shapes tend to be focused on their Areas or Perimeters.

Whereas questions on 3D shapes tend to concern their Volumes.

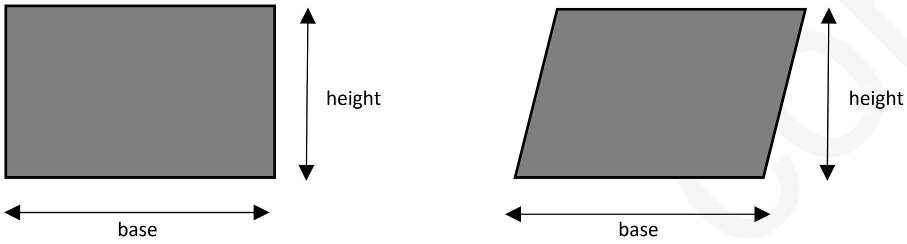
Areas & Perimeters:

Area refers to the coverage or space within a shape.

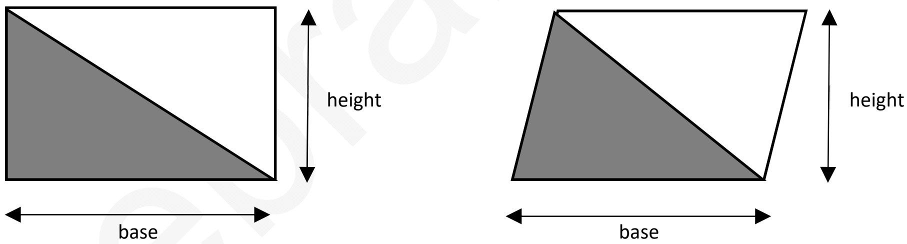
Perimeter is the distance surrounding the specified shape.

We should always remember the equation for calculating a quadrilateral (four-sided shape) as being equal to it's base × height.

$$\text{Area of quadrilateral } (m^2) = \text{base } (m) \times \text{height } (m)$$



Conversely, a triangle can be obtained by cutting a quadrilateral into one-half (from one corner to the opposite corner). Thus the equation for an Area of a triangle is half of a quadrilateral:

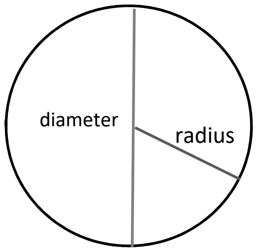


$$\text{Area of Triangle } (m^2) = \frac{\text{base } (m) \times \text{height } (m)}{2}$$

Notice the units-of-measurement which we included in the equations and workings. For example, cm x cm = cm^2, km x km = km^2…etc. The units just like the number coefficients and variables in algebraic expressions are also subject to the same rules with have seen in SelfStudy guide A3 – Algebra & Equations! Please remember this.

The 2-superscript is called 'squared'. m^2 is metres squared. km^2 is kilometres squared. the 'squared' measurement is indicative of Area!

There is one uniform 2D shape that has only ONE side…this is the circle. Let us introduce two more words which are used to define circles:



Diameter ('**D**') is the distance between one end to the other end of the circle passing through the centre.

Radius ('**r**') is distance from the centre to edge of the circle.

The following are important relationships between the parameters of circles.

D = 2r

Area of circle = πr^2

Circumference (perimeter of circle) = $2\pi r$ = $D\pi$

π (pronounced "Pi") is a constant, a fixed number approximately equal to 3.141592…

[Sometimes also approximated by the fraction $\frac{22}{7}$]

Example:

Find the Area and Perimeter of a rectangle whose length is 23cm and whose width is 28cm?

Steps:

(1) **ALWAYS** make sure you check the **units** are at the scale you desire before calculating. If necessarily, you may need to convert the units into same form or as requested by the question.

(2) As we know Area of quadrilateral = base × height
Therefore Area, 23cm × 28cm = **644cm^2**

(3) Perimeter = 23cm + 28cm + 23cm + 28cm = 2 × (23cm + 28cm) = 2 × 51cm = **102cm**

Your answer is not entirely correct unless the Units of your answer is also correct!

Example:

Using the same rectangle given above, Tom decided to cut the rectangle across its diagonal corners to create two triangles. Determine the area of each triangle.

Steps:

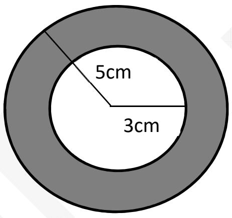
(1) Area of Triangle = $\frac{\text{base (m)} \times \text{height (m)}}{2}$

(2) $\frac{23\text{cm} \times 28\text{cm}}{2}$ = **322cm^2**

The Area of both triangles will be identical given both triangles are mirror images of each other.

Example: Work out the area of the grey ring:

Assume π = 3.14



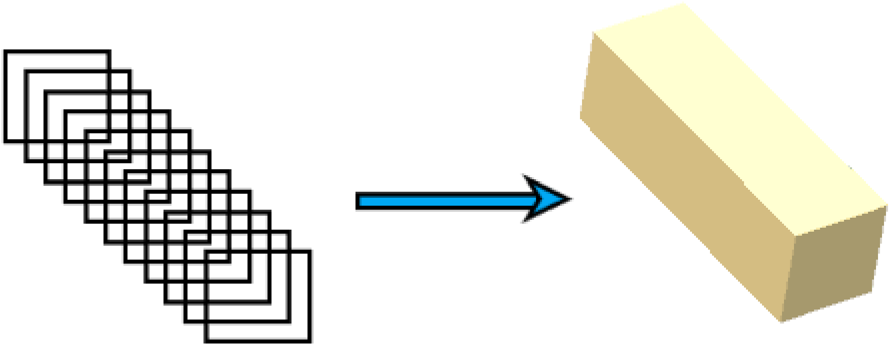
Steps:

(1) We can work out the area of the larger circle (with 5cm radius) and then deduct away the area of the smaller circle within it (with 3cm radius).
(2) Area of circle = πr^2
(3) Area of larger circle = 25π cm^2
Area of smaller blue circle = 9π cm^2
Area of Ring = large circle – smaller circle = $25\pi - 9\pi = 16\pi$ cm^2

16cm$^2 \times 3.14$ = **50.24cm^2**

Volumes:

For simple 3D shapes such as a cube, cuboid or cylinder. Their shape is essentially built up from multi-layered 2D shapes.



Hence the equations which define the Volume of these shapes can be viewed as being derived or built from their 2D Area formulas…we only add one more dimension to make it into 3D!

Volume of Cube & Cuboid (m^3) = base (m) × height (m) × width (m)

Volume of Triangular Prism (m^3) = $\dfrac{\text{base (m)} \times \text{height (m)}}{2}$ × width (m)

Volume of cylinder (m^3) = πr^2 (m^2) × height (m)

The 3-superscript is called 'cubed'. m^3 is metres cubed. cm^3 is centimetres cubed. the 'cubed' measurement is indicative of Volume!

Example:

Gravy stock cubes are 2cm × 2cm × 2cm in size. They are packed into a box which is 20cm × 18cm × 26cm. What is the volume of a single stock cube? volume of one box? And how many stock cubes are needed to fill a box?

Steps:

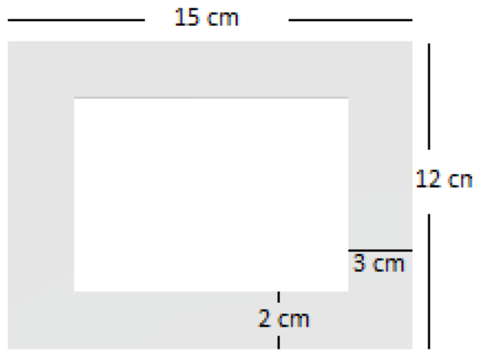
(1) Volume:
   a. Volume of stock cube = 2cm × 2cm × 2cm = **8cm^3**
   b. Volume of cuboid box = 20cm × 18cm ×26 cm = **9,360cm^3**
      (use your long-multiplication skills. See SelfStudy Guide A1)

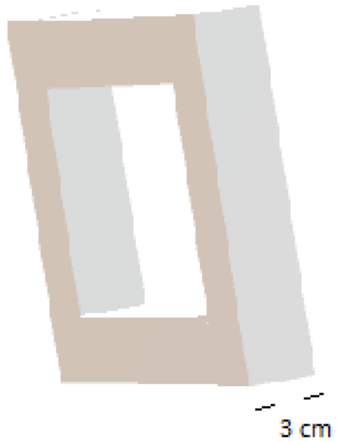
(2) In your mind, test that the stock cubes can line the box perfectly without leaving any spare gaps. In this case, the size of the cubes will pack into the box completely.

Given the above circumstances, the number of stock cubes needed to fill the box will be the volume of the box divided by the volume of a stock cube. $9{,}360cm^3 \div 8cm^3$ = **1,170 cubes.**

Example:

Work out the volume of the following picture frame. Side view of the picture frame have been presented to help with visualising.





Step:

(1) It's easier to work out the 2D area before multiplying by the width to get the volume.

(2) From the front view of the picture frame, we can break the picture frame into two sets of rectangles (a & b).

a. $12cm \times 3cm = 36cm^2$

b. $(15cm - 3cm - 3cm) \times 2cm = 18cm^2$

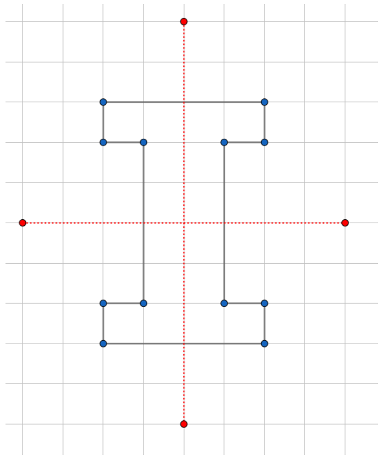
Cross-sectional area of frame = $(36cm^2 + 18cm^2) \times 2 = 108cm^2$

Volume of frame = cross-sectional area x width

$\therefore$ $= 108cm^2 \times 3cm^2 = \mathbf{324cm^3}$

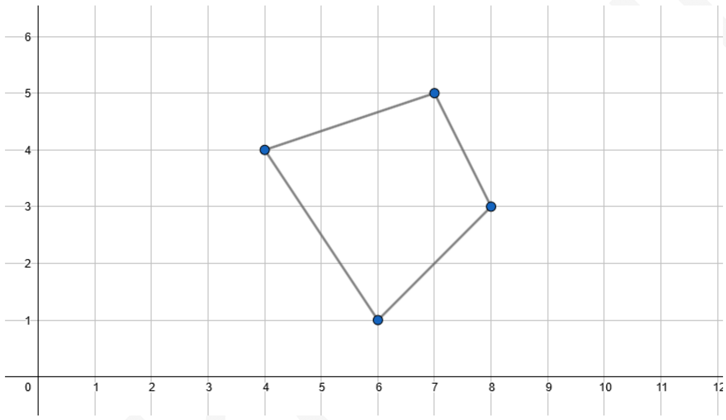
Lines of Symmetry & Rotational Symmetry:

Line of symmetry: If you fold a piece of A4 paper into half. You'll get two areas of equivalent size. The folded-line that we have created is called a line of symmetry. Placing a mirror on this line will also yield back your original shape through reflection.



As can be seen from this picture of the letter "I", we can say it has TWO lines of symmetry (drawn in red).

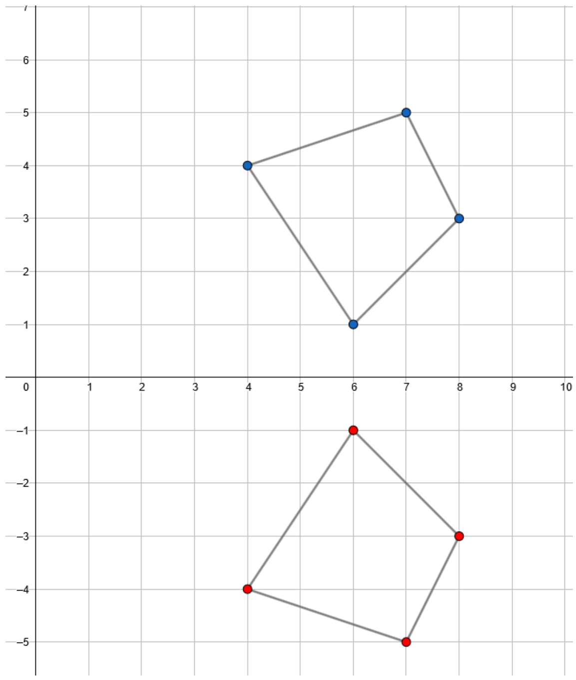
Example: Reflect the below picture through the x axis.



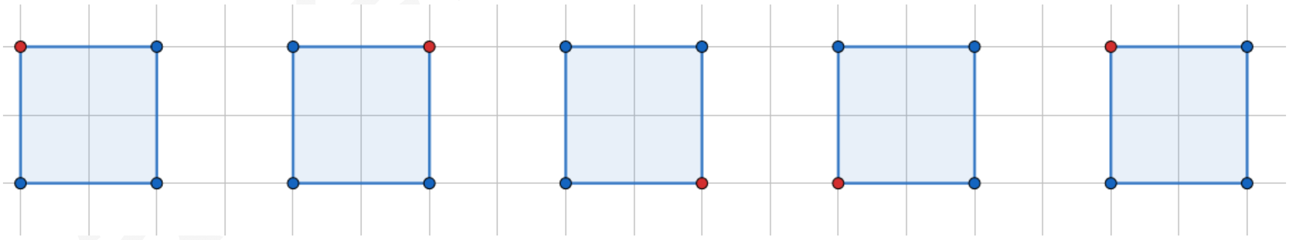
Steps:

(1) The question is explicit that it wants us to use the x-axis as the line of symmetry. The x-axis is the horizontal line on the graph where y = 0 (therefore the lowest line visible in the above illustration).

(2) Reflect the object using this line. You may count squares to pin-point corners equi-distant from the line of symmetry. You may use your imagination and check that you have created a mirror image. The result should be illustrated as follows:



Rotational Symmetry: This is symmetry around a single point *within* the shape (as opposed to a line). Using a square as an example: If a square is rotated about its centre, we may not be able to distinguish whether any rotation had occurred under four positions. However, if we were to label a corner and track its movement, the square will get back into its original position after FOUR 90° rotations in the same direction. WE describe this square as having a rotational symmetry of the 'order-four'.

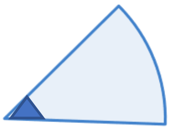
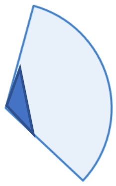
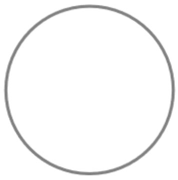
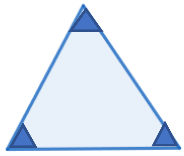


A rectangle will have rotational symmetry of the order-Two. A perfect triangle 'order-Three'. An isosceles triangle 'order-One'. Try and see whether you can visualise these in your mind!

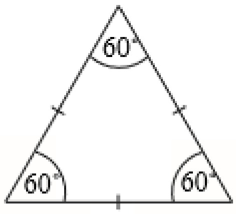
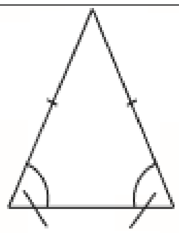
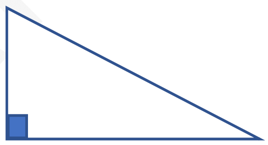
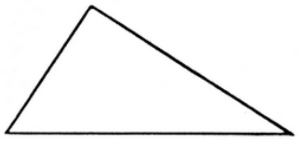
Introducing Angles:

Two-lines which converge together form an apex (a point); the *spread* between these two-lines is called an 'angle°'… this spread can be measured using an instrument such as a protractor.

Below are some common angles found in popular shapes.

				
Right-Angle = 90° The sum of *all* angles *inside* a Quadrilateral = 360°	**Acute Angle** = Less than 90°	**Obtuse Angle** = Greater than 90°	A full Circle is 360°	The sum of *all* angles *inside* a Triangle = 180°

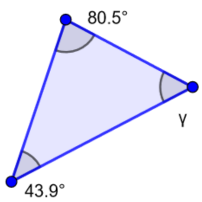
Below are the different triangle types you will come across:

			
Equilateral Triangle: where all three corners and sides are equal to another.	**Isosceles Triangle:** where two corners and two sides are equal to one another.	**Right Angle Triangle:** where one corner is 90°	**Scalene Triangle** where there are no equivalent angles or sides.

Please study the above patterns carefully and memorise them through practice. They will enable you to solve geometric questions.

Example:

Find the size of the angle marked with the letter Y in the triangle below:



Steps:

(1) The sum of *all* angles *inside* a Triangle = 180°

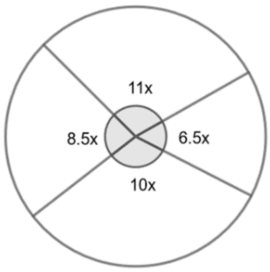
Thus solve this equation and rearrange to target Y (refer to SelfStudy Guide A3 – Algebra & Equations if necessary).

$$43.9° + 80.5° + Y = 180°$$

$$\Longrightarrow \quad Y = 180° - 43.9° - 80.5°$$

$$\therefore \quad \mathbf{Y = 55.6°}$$

Example: Determine 'x' and the angles of all four sectors in the below circle:



Steps:

(1) This question combinations your algebraic skills with geometric knowledge!! Awesome! The key starting point is being aware that a full circle is 360°.

Thus $\quad 11x + 6.5x + 10x + 8.5x = 360°$

Factor out x° $\quad x(11 + 6.5 + 10 + 8.5) = 360°$

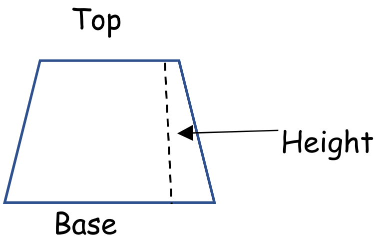
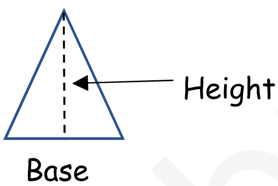
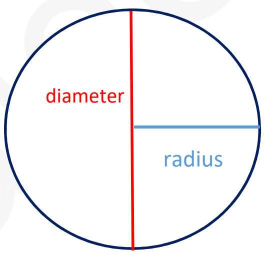
$\Longrightarrow \quad 36x = 360°$

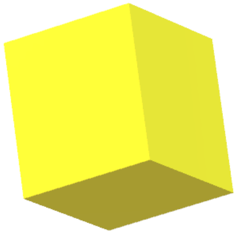
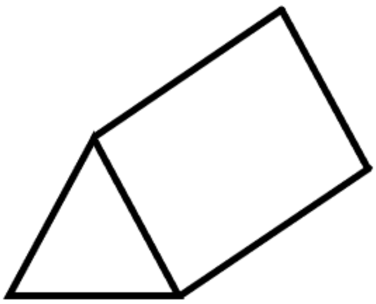
$\therefore \quad \mathbf{x = 10°}$

(2) The angles of each sector must therefore be:
110°, 65°, 100° and 85°.

APPENDIX - formulas

To consolidate your knowledge, you must practise, practise and practise! Enquire about our classroom & webinar courses or Question & Answer materials…visit us at www.algebrains.com

 Square, Rectangles, Rhombus…	Area of quadrilateral (square, rectangle, rhombus or parallelogram) = base × height
 Trapezium	Area of trapezium $= \frac{(\text{top length} + \text{base})}{2} \times \text{height}$
 Triangle	Area of Triangle $= \frac{\text{base} \times \text{height}}{2}$
 Circle	Diameter (D) = radius (r) × 2 Area of circle = πr^2 Circumference = $2\pi r$ = $D\pi$

  Cube cuboid	Volume of Cube & Cuboid = base × height × width
 Cylinder	Volume of cylinder = $\pi r^2 \times$ height
 Prism	Volume of Triangular Prism $= \dfrac{\text{base} \times \text{height}}{2} \times$ width

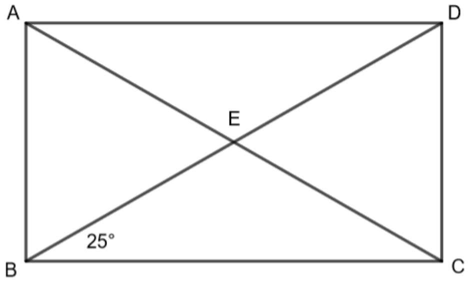
Practice Questions:

Calculate & show workings…

Question 1:

The diagram below shows a rectangle with its diagonals drawn.

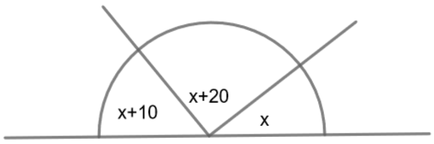
Angle DBC has been marked with an angle of 25°.



(a) List all other angles that have the same angle as angle DBC
(b) Find the size of all other angles in the diagram.

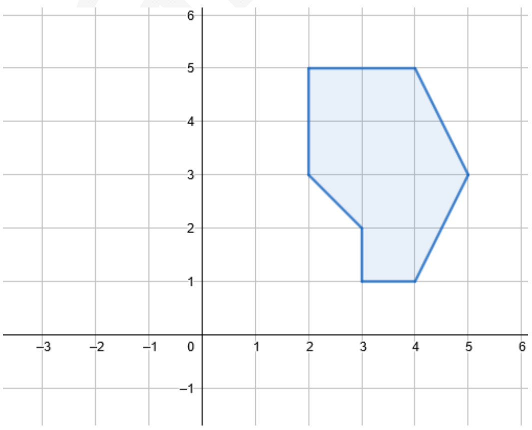
Question 2:

Determine 'x' in the following diagram and then the angles of each of the three sections.



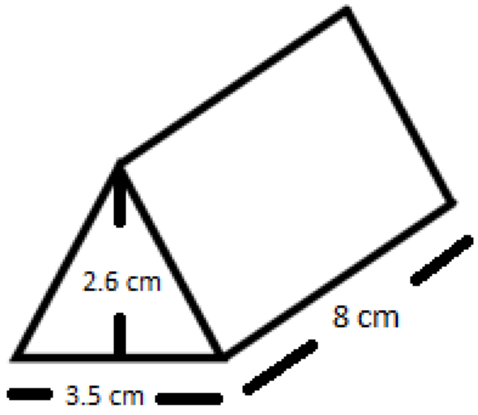
Question 3:

Reflect this picture along the y axis.



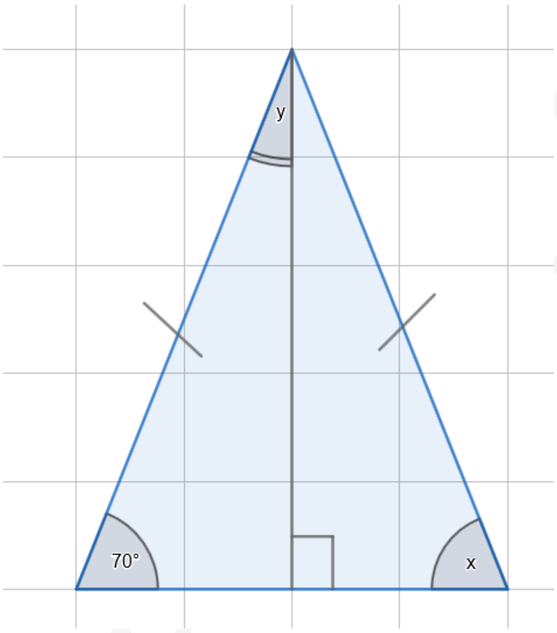
Question 4:

Work out the volume of the prism



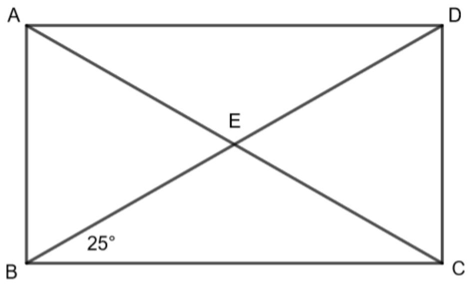
Question 5:

Work out the missing angles



Answers & Model Workings:

Question 1:
The diagram below shows a rectangle with its diagonals drawn.
Angle DBC has been marked with an angle of 25°.



(a) List all other angles that have the same angle as angle DBC
(b) Find the size of all other angles in the diagram.

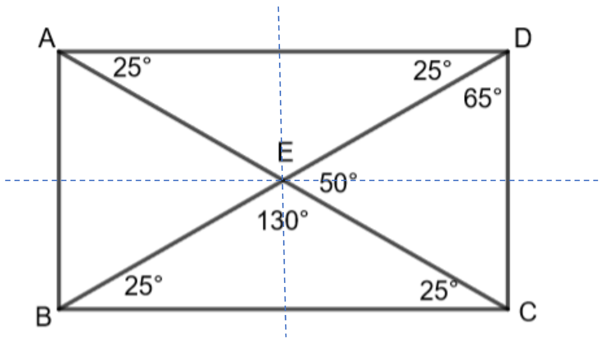
(1) As the rectangle is symmetrical through the two-lines of symmetry (see below), we can use this fact to identify the other 25° angles easily using mirror images.

[Note, in labelling angles, the three letters represent how the angle is drawn, but the middle letter represent the actual angle referring to].

Angles ACB, DAC, ADB all have same angle of 25°.

(2) From the diagram we can see two sets of isosceles triangles embedded within this rectangle. Sets 'AED & BEC' and 'CED & AEB'.

Isosceles triangles have a pair of identical angles either side of its line-of-symmetry. Plus we can use the rule 'sum of all internal angles in a triangle equals 180°.

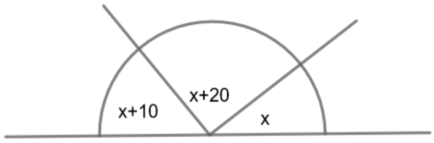


As each corner for a rectangle is a right angle of 90°, we can then deduce the angle such as EDC to 65°.
Hence we can work out angle CED which is an isosceles as 180° – 65° – 65° = 50°
And angle AED as 180° – 25° – 25° = 130°
Overall **50° = DEC = AEB**
130° = AED = BEC
25° = DBC = ACB = ADB = DAC
65° = EDC = ECD = EBA = EAB

Question 2:

A straight line is equivalent to half a circle, which has an angle of 180°.

Work out 'x' in the following diagram and then the angles of each of the three sections.



We know the sum of the angles of all three-sectors must equal to 180°.

Therefore $x + 10° + x + 20° + x = 180°$

$3x + 30° = 180°$

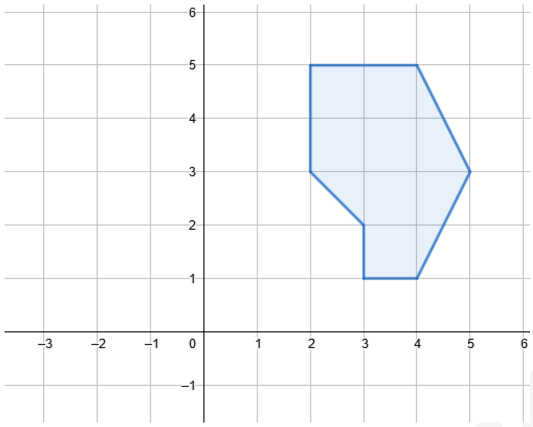
$3x = 150°$

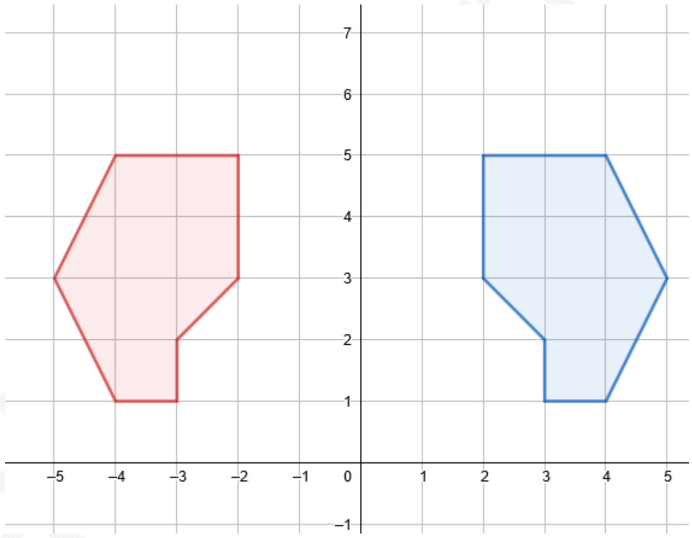
$x = 50°$

So each section would have an angle of 60°; 70° and 50°

Question 3:

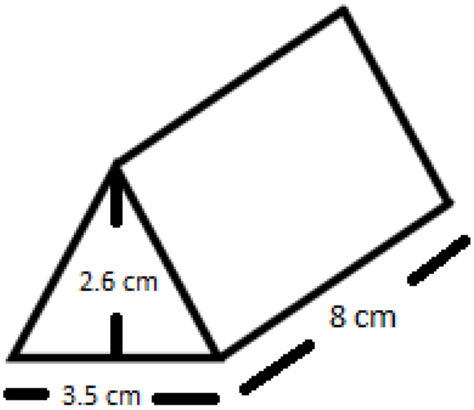
Reflect this picture along the line given on the graph.





Question 4:

Work out the volume of the prism



As mentioned in the text, it's easier to determine the area before multiplying this by the width to derive volume.

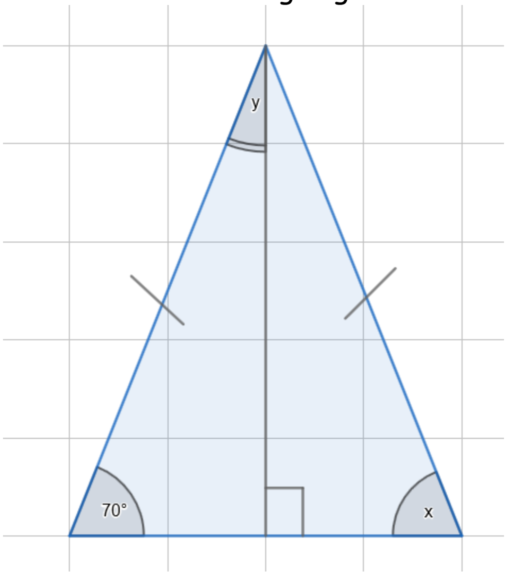
Prism has cross sections of triangles, hence to determine area of triangle, we use,

$\frac{\text{base} \times \text{height}}{2}$ which is...

$3.5\text{cm} \times 2.6\text{cm} \div 2 = 4.55\text{cm}^2$

We then multiply 4.55cm^2 against the width 8cm to get volume...

$4.55\text{cm}^2 \times 8\text{cm} =$ **36.4cm^3**

Question 5: Work out the missing angles 	Hopefully you will be able to identify this is an isosceles triangle that has been cut into half (with 1 line of symmetry). As by definition of the isosceles, the two corner angles either side of the line of symmetry are same. Therefore, you can simply read-off that **$x = 70°$**. We now need to determine 'y', but please note that y is the angle of the smaller triangles after the isosceles have been sliced in half. We are also given a right-angle symbol in the diagram, knowing it represents 90°, together with the other known angle being 70°. We now have enough information to derive the missing angle y. Angle $y = 180° - 90° - 70° = \mathbf{20°}$ **Overall, $x = 70°$ and $y = 20°$**