Algebrains

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SelfStudy Series

A1 – *Long* addition, subtraction, multiplication & division

version 1.04

Contents

PREFACE

Dear parents, guardians and teachers. Thank you for purchasing this study guide directly from algebrains.com. Our SelfStudy guides are available exclusively from algebrains.com (or from our offices) and have been priced to encourage greater accessibility from many students and their families who will benefit from our content. By purchasing directly, you are also contributing and supporting our mission in strengthening the delivery of Maths & Financial Education to children & young-adults in Britain (and throughout the world).

Our SelfStudy series have been written for students as a reference to teach them how to tackle mathematical challenges via step-by-step illustrations. Our materials have been designed to help parents to easily understand the workings too, to help you coach your child.

We have kept the content as concise and as pictorial as possible…so that our examples are easy to follow…therefore easy to understand and apply! We have also decided not to distract the students with elaborate colours as their exam papers will be in black & white.

Should you choose to complement your child's study with our classroom or webinar sessions, your child will also have access to additional illustrated workings for all questions that we shall practise.

Regardless of your child's level, whether a beginner or advanced…we firmly believe that our learning materials coupled with frequent practise will transform your young ones into numerically competent magicians!

Good luck and enjoy learning!

Ying & Jerry

Long-Addition:

Let's introduce the number cupboard, which is essentially our numbering system where every number can be broken down between its thousands, hundreds, tens and units.

Billions			Millions			Thousands			Hundreds	Tens	Units
x	x	x ,	x	x	x ,	x	x	x ,	x	x	x .

Only ONE number can be placed into each cell (row & column combination) of the cupboard. For example 1 + 1 could be presented in the cupboard as below:

	Units
	1
+	1
	2

This columnar format should be practiced until you are comfortable with working with this system.

Example: 1 + 9

	Tens[1]	Units
		1
+		9
	1	0

Steps:

(1) Place the 1 (unit) into the top row under Units column
(2) Place the 9 (another unit) into the second row of Unit column
(3) Remember to mark "+" on the left to remind others we are calculating an addition
(4) numbers are added from columns right to left, in this example we only have one column (Units).
(5) 1 + 9 = 10. We now need to split this 10 into Units and Tens in the answer row (bottom row):
    i) we need to place the 0 (unit) of the 10 in the Units column and
    ii) place the 1-prefix (of the 10) in reserve into the Tens cupboard.

(6) As there is nothing else held in the Tens cupboard, we bring the 1 held in reserve and place this in the answer row. You have demonstrated through your workings how you arrived at the answer 10!

Let's try a bigger number using the same method:

Example: 28 + 56

Steps:

(1) Place number 28 into the relevant first row cupboards (Tens & Units)

(2) Do the same for number 56 in the second row. Remember to mark "+" on the left to remind others of your intention (to add).

	Tens [1]	Units
	2	8
+	5	6

(3) Now add each column from <u>right to left</u>.

i) Units: 8 + 6 = 14, so put 4 (of the 14) into the answer row under Units.

ii) Tens: place the 1-prefix (of the 14) into reserve of the Tens cupboard

	Tens [1]	Units
	2	8
+	5	6
		4

iii) Now add all the numbers in the Ten Cupboards INCLUDING numbers held in reserve: I prefer top-down… 1 + 2 + 5 = 8

	Tens [1]	Units
	2	8
+	5	6
	8	4

iv) Place the 8 into the answer row of the Tens cupboard. 84 is your final answer!

One more example just to make sure you have understood this method. It does not matter how large, or how many numbers to add …the method is the same.

Example: 253 + 98 + 21

Steps:

(1) Start by placing the question into the number cupboard as below

	Hundreds	Tens	Units
	2	5	3
		9	8
+		2	1

(2) Add the columns from right to left.

i) Units: 3 + 8 + 1 = 12. Leave the 2 in the answer row and carry the 1-prefix to the Tens.

	Hundreds	Tens ^{1}	Units
	2	5	3
		9	8
+		2	1
			2

ii) Tens: 1 + 5 + 9 + 2 = 17. Leave the 7 in the answer row and carry the 1-prefix into the next column, the Hundreds.

	Hundreds ^{1}	Tens ^{1}	Units
	2	5	3
		9	8
+		2	1
		7	2

iii) Hundreds: 1 + 2 = 3. Place the 3 in the answer row

	Hundreds ^{1}	Tens ^{1}	Units
	2	5	3
		9	8
+		2	1
	3	7	2

Well done for showing your workings to arrive at the final answer of 372!

Long-Subtraction:

Subtraction is the twin of addition

Under long-addition we have learnt to carry the first digit for numbers greater than 9 to the left column(s). In contrast, for long-subtraction, we will learn to borrow digits from the left column(s).

Let's start with a simple subtraction

Example: 9 – 5

	Units
	9
-	5
	4

Steps:

(1) Place 9 (unit) on top row of Units cupboard
(2) Place 5 (another unit) in the second row
(3) Remember to mark "-" on the left to remind others we are calculating a subtraction
(4) numbers are subtracted, working from columns right to left, in this example, we only have one column (Units).
(5) 9 – 5 = 4

Example: 43 – 7

Steps:

(1) Place number 43 into the relevant first row cupboards (Tens & Units)

(2) Do the same for number 7 in the second row. Remember to mark "-" on the left to remind others of your intention (to subtract).

	Tens	Units
	4	3
-		7

(3) Work on each column from right to left.
   i) Units: 3 – 7. Mmmm….3 take away 7 will give us a negative number…we don't want that, so we borrow a 1-prefix from the left column.
      - The 4 becomes a 3 (in the Tens cupboard), 1 less, and…

- The 3 (in Units) now becomes 13 having borrowed a 1-prefix!

	Tens	Units
	~~4~~ 3	1 3
-		7

ii) Check the columns to the left to see whether there are any numbers in the second row greater than its corresponding number above it in the first row. If yes, then ask the top row number to borrow a 1-prefix from its left top row neighbour.

(4) Now subtract the second row number from the top row per cupboard

i) Units: 13 – 7 = 6

ii) Tens: 3

	Tens	Units
	~~4~~ 3	1 3
-		7
	3	6

36 is your final answer!

One more example just to make sure you have understood this method. It does not matter how large are the numbers …the method for subtracting one number from the other is the same.

Example: 517 – 89

Steps:

(1) Start by placing the question into the number cupboard as below

	Hundreds	Tens	Units
	5	1	7
-		8	9

(2) Check columns from right to left to see whether the second row number is greater than the top row number. If yes, get the top number to borrow a 1-prefix from its left neighbour.
In this example, both the Tens and Units Cupboard need to borrow

i) Units: Working on the right cupboard first before moving to the left…7 is too small versus the 9. The 7 borrows a 1-prefix from its left neighbour 1; reducing the latter to zero.

	Hundreds	Tens	Units
	5	~~1~~ 0	1 7
-		8	9

ii) Tens: The 0 is too small versus the 8. The 0 borrows a 1-prefix from its left neighbour 5; reducing the latter to 4.

	Hundreds	Tens	Units
	~~5~~ 4	1 0	1 7
-		8	9

(3) Now that the top numbers are greater or equal to their bottom numbers, you may subtract the numbers in each column from right to left:

	Hundreds	Tens	Units
	~~5~~ 4	1 0	1 7
-		8	9
	4	2	8

Well done for showing your workings to arrive at the final answer of 428!

Long-Multiplication:

As a starting point, we are going to assume you are already fluent with your timetables. If not, refer to the Appendix and learn the tables vigorously. You must strengthen your roots!!!

Below, we are going to show you how to use this knowledge to derive the multiplication of bigger numbers.

Example: 32 x 14

Steps:

(1) Place the numbers into the cupboard as demonstrated below

	Hundreds	Tens	Units
		3	2
x		1	4

(2) As with long-addition and long-subtraction, start from right to left:

i) Start from the second row right number…number 4 (of 14)…and multiply this number against each digit in the top row (like spokes in wheels with 4 being the core) starting from the top digit on the right.

e.g. 4 x 2 = 8 (place the 8 in the answer column)

4 x 3 = 12 (place the 2 in the answer column and carry the 1-prefix to the next left column)

	Hundreds	Tens	Units
		3	2
x		1	4
	1	2	8

ii) Repeat this for the next left second row number 1 (of 14). Multiply this number against each digit in the top row starting from the top right digit and work towards your left.

e.g. 1 x 2 = 2 (place the 2 directly in the same column as the core 1)

1 x 3 = 3 (place the 3 in the next left column)

	Hundreds	Tens	Units
		3	2
x		1	4
	1	2	8
	3	2	

(3) Sum the last two rows including any carried over numbers column-by-column starting from right to left.

	Hundreds	Tens	Units
		3	2
x		1	4
	1	2	8
+	3	2	
	4	4	8

Well done for showing your workings to arrive at the final answer of 448!

Let's do one more example just to make sure you have understood this method. It does not matter how large are the numbers …the method for multiplying TWO string of numbers is the same.

Example: 285 x 79

Steps:

(1) Place the numbers into the cupboard as demonstrated below (smallest number in second row)

	Thousands	Hundreds	Tens	Units
		2	8	5
x			7	9

(2) Start from the second row right number…number 9 (of 79) …and multiply this number against each digit in the top row (like spokes in wheels with 9 being the core) starting from the top digit on the right.

e.g. 9 x 5 = 45 (place the 5 in the answer column, carry the 4 to the next left column)

9 x 8 = 72 (place the 2 in the answer column and carry the 7-prefix to the next left column)

9 x 2 = 18 (place the 8 in the answer column, carry the 1 to the next left column)

Thousands	Hundreds [7]	Tens [4]	Units
	2	8	5

x			7	9
	1	8	2	5

(3) Repeat this for the next left second row number 7 (of 79). Multiply this number against each digit in the top row starting from the top right digit and work towards your left.

e.g. 7 x 5 = 35 (place the 5 in the answer column, the same column as the core 7, carry over the 3 to the next left column)

7 x 8 = 56 (place the 6 in the next answer column on the left, carry over the 5 to the next left column)

7 x 2 = 14 (place the 4 in the next answer column, carry over the 1 to the next left column)

	Thousands	Thousands ^{5}	Hundreds $^{7+3}$	Tens ^{4}	Units
			2	8	5
x				7	9
		1	8	2	5
	1	4	6	5	

(4) Sum the last two rows including all carried over numbers column-by-column starting from right to left. Remember what you have learnt in Long-addition…you may need to carry over more numbers to the next columns on the left.

	Thousands ^{1}	Thousands $^{5+2}$	Hundreds $^{7+3+1}$	Tens ^{4}	Units
			2	8	5
x				7	9
		1	8	2	5
+	1	4	6	5	
	2	2	5	1	5

Well done for showing your workings to arrive at the final answer of 22,515!

Long-Division:

Division is the opposite of multiplication.

The way we present division is slightly different from addition, subtraction *and* multiplication, however the principle of having columns of units, tens, hundreds…etc. are exactly same.

Example: 20 ÷ 9

This can be alternatively presented in a fractional format $\frac{20}{9}$ or just 20 / 9…but for our workings we shall show it in this format.

$$9 \overline{)\ 2 \quad 0}$$

You will soon see that your answer will be placed at the top and the remainders (if any) at the bottom of your workings.

Steps:

(1) For division, we shall work from the opposite-end…from left to right

i) 9 cannot be squeezed into a smaller number 2 (on the right), so we shall consider whether 9 can be squeezed into the next larger number 20 (by considering the 0 on the right of number 2)?

ii) 2 nines can fit into 20, so we place the factor 2 in the upper answer row above the last considered number, the 0:

$$\begin{array}{r} \quad 2 \\ 9 \overline{)\ 2 \quad 0} \end{array}$$

iii) 2 nines = 18, so we write 18 below the 20 and use our long-subtraction skills to deduct 18 from 20 to determine any remainder:

$$\begin{array}{r} \quad 2 \\ 9 \overline{)\ 2 \quad 0} \\ -\ 1 \quad 8 \\ \hline \quad 2 \end{array}$$

20 ÷ 9 is equal to '2 remainder 2'.

(2) The remainder is like the excess pork-trimmings, and for some questions we may stop here if the remainder makes real physical sense…for example: how many sweets remain if 20 sweets are shared equally amount 9 children?

For some other applications where precision is required, we can continue with the long-division to convert the remainder into a decimal.

i) We do this by further dividing the remainder: The 9 cannot fit into the remainder 2 so we place an imaginary 0 on its right to make 20.

ii) 2 nines can now be squeezed into the 20, so we place the factor 2 in the next column in the top answer row. Notice that I have placed a decimal place in front of this 2 as we have now shifted to magnitudes smaller than Units!....in otherwords, we are moving into decimals.

```
      2 . 2
   ┌──────
9  │ 2  0
-  1  8
   ──────
      2  0
```

iii) 2 nines is 18, so again deduct the 18 from the 20 to arrive at the new remainder.

```
      2 . 2
   ┌──────
9  │ 2  0
-  1  8
   ──────
      2  0
   -  1  8
   ──────
         2
```

Notice that the remainder is again 2! This division will continue to repeat itself…

… 20 ÷ 9 in decimals is = 2.2222222

Example: 3567 ÷ 8

$$8 \overline{)\,3\;5\;6\;7}$$

(1) 8 cannot be squeezed into 3, so we consider the next number on the right to form 35.

(2) 8 can squeeze into 35 only 4 times, so we place the factor 4 in the answer row above the last number considered, the 5:

$$\begin{array}{r} \;\;\;4 \\ 8 \overline{)\,3\;5\;6\;7} \end{array}$$

(3) 8 x 4 = 32, so deduct 32 from 35 to derive the remainder 3:

$$\begin{array}{r} \;\;\;4 \\ 8 \overline{)\,3\;5\;6\;7} \\ -\;3\;2 \\ \hline \;\;\;3 \end{array}$$

(4) Now drop the remaining digits 6 and 7 next to the remainder to arrive at the next number to divided into:

$$\begin{array}{r} \;\;\;4 \\ 8 \overline{)\,3\;5\;6\;7} \\ -\;3\;2 \\ \hline \;\;\;3\;6\;7 \end{array}$$

(5) Repeat the division method: Can 9 be squeezed into 3? No…how about 36? How many 8's can be squeezed into 36? Four …place the factor 4 into the top answer row directly above the last number considered, the 6 (of 36):

$$\begin{array}{r} \;\;\;4\;4 \\ 8 \overline{)\,3\;5\;6\;7} \\ -\;3\;2 \\ \hline \;\;\;3\;6\;7 \end{array}$$

(6) Derive the remainder by, subtracting 32 [8 x 4] from the 36:

```
      4  4
   ┌──────
8  │ 3  5  6  7
-  │ 3  2
   ──────
      3  6  7
   -  3  2
      ──────
         4
```

(7) Now drop the remaining digit 7 next to the remainder to arrive at the next number to divided into:

```
      4  4
   ┌──────
8  │ 3  5  6  7
-  │ 3  2
   ──────
      3  6  7
   -  3  2
      ──────
         4  7
```

(8) Repeat the process. Can 8 be squeezed into a 4? No…how about 47? How many times can 8 be squeezed into 47…5 times… place the factor 5 into the answer row:

```
      4  4  5
   ┌─────────
8  │ 3  5  6  7
-  │ 3  2
   ──────
      3  6  7
   -  3  2
      ──────
         4  7
```

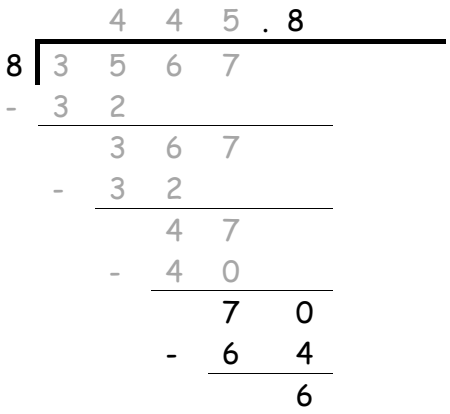
(9) Derive the remainder by deducting 40 [8 x 5] from 47:

```
      4  4  5
   ┌─────────
8  │ 3  5  6  7
-  │ 3  2
   ──────
      3  6  7
   -  3  2
      ──────
         4  7
      -  4  0
         ──────
            7
```

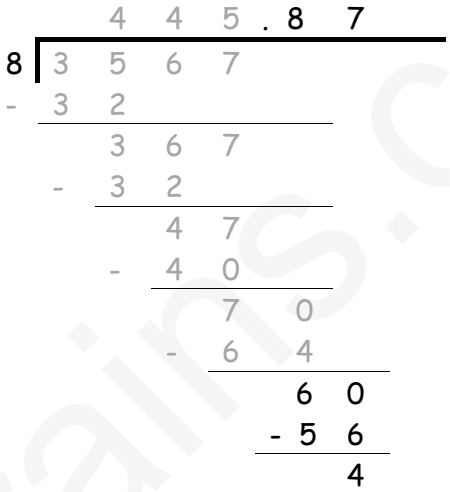
The answer of *3567 ÷ 8* is '445 remainder 7'.

However, one can continue to repeat the division to drill further to derive the decimal places:

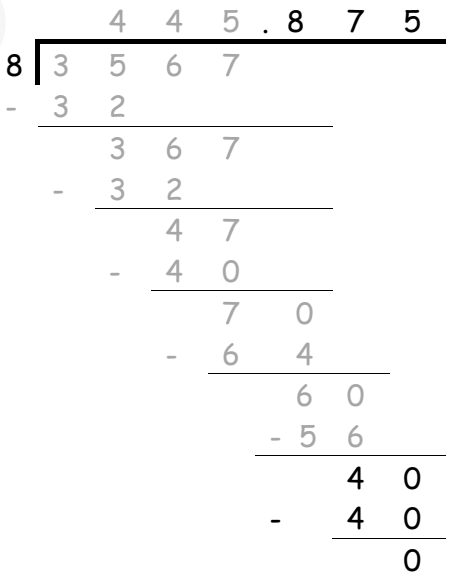
1-decimal place:



2-decimal places:



3-decimal places:



Well done for getting this far…after many iterations…you have shown full workings to arrive at the final answer of 445.875!

APPENDIX

Multiplication tables. Print this page and stick it next to your bed… Please learn the below tables off-by-heart ☺

1 x 1 = 1
1 x 2 = 2
1 x 3 = 3
1 x 4 = 4
1 x 5 = 5
1 x 6 = 6
1 x 7 = 7
1 x 8 = 8
1 x 9 = 9
1 x 10 = 10
1 x 11 = 11
1 x 12 = 12

2 x 1 = 2
2 x 2 = 4
2 x 3 = 6
2 x 4 = 8
2 x 5 = 10
2 x 6 = 12
2 x 7 = 14
2 x 8 = 16
2 x 9 = 18
2 x 10 = 20
2 x 11 = 22
2 x 12 = 24

3 x 1 = 3
3 x 2 = 6
3 x 3 = 9
3 x 4 = 12
3 x 5 = 15
3 x 6 = 18
3 x 7 = 21
3 x 8 = 24
3 x 9 = 27
3 x 10 = 30
3 x 11 = 33
3 x 12 = 36

4 x 1 = 4
4 x 2 = 8
4 x 3 = 12
4 x 4 = 16
4 x 5 = 20
4 x 6 = 24
4 x 7 = 28
4 x 8 = 32
4 x 9 = 36
4 x 10 = 40
4 x 11 = 44
4 x 12 = 48

5 x 1 = 5
5 x 2 = 10
5 x 3 = 15
5 x 4 = 20
5 x 5 = 25
5 x 6 = 30
5 x 7 = 35
5 x 8 = 40
5 x 9 = 45
5 x 10 = 50
5 x 11 = 55
5 x 12 = 60

6 x 1 = 6
6 x 2 = 12
6 x 3 = 18
6 x 4 = 24
6 x 5 = 30
6 x 6 = 36
6 x 7 = 42
6 x 8 = 48
6 x 9 = 54
6 x 10 = 60
6 x 11 = 66
6 x 12 = 72

7 x 1 = 7
7 x 2 = 14
7 x 3 = 21
7 x 4 = 28
7 x 5 = 35
7 x 6 = 42
7 x 7 = 49
7 x 8 = 56
7 x 9 = 63
7 x 10 = 70
7 x 11 = 77
7 x 12 = 84

8 x 1 = 8
8 x 2 = 16
8 x 3 = 24
8 x 4 = 32
8 x 5 = 40
8 x 6 = 48
8 x 7 = 56
8 x 8 = 64
8 x 9 = 72
8 x 10 = 80
8 x 11 = 88
8 x 12 = 96

9 x 1 = 9
9 x 2 = 18
9 x 3 = 27
9 x 4 = 36
9 x 5 = 45
9 x 6 = 54
9 x 7 = 63
9 x 8 = 72
9 x 9 = 81
9 x 10 = 90
9 x 11 = 99
9 x 12 = 108

10 x 1 = 10
10 x 2 = 20
10 x 3 = 30
10 x 4 = 40
10 x 5 = 50
10 x 6 = 60
10 x 7 = 70
10 x 8 = 80
10 x 9 = 90
10 x 10 = 100
10 x 11 = 110
10 x 12 = 120

11 x 1 = 11
11 x 2 = 22
11 x 3 = 33
11 x 4 = 44
11 x 5 = 55
11 x 6 = 66
11 x 7 = 77
11 x 8 = 88
11 x 9 = 99
11 x 10 = 110
11 x 11 = 121
11 x 12 = 132

12 x 1 = 12
12 x 2 = 24
12 x 3 = 36
12 x 4 = 48
12 x 5 = 60
12 x 6 = 72
12 x 7 = 84
12 x 8 = 96
12 x 9 = 108
12 x 10 = 120
12 x 11 = 132
12 x 12 = 144

Practise Questions

Calculate & show workings…

Question 1:

a) 25 + 87
b) 102 – 79
c) 56 × 75
d) 5.6 × 7.5
e) 216 ÷ 180

Question 2:

The teacher asked David for a random number from 1 to 5,000 and Venessa another number from 1 to 2,500.

David chose number 4,096 and Venessa chose number is 1,024.

What's the total of their numbers?

Question 3:

The teacher then asked what's the difference between the number given by David and that of Vanessa's.

Question 4:

Jess took £10.00 with her to the local shop. She purchased two packet of crisps costing 74p each, one bottle of milk costing £1.48, three cans of drinks costing 63p each and two packs of ready-meals costing £2.38 each. How much change did she get back?

Answers & Model Workings

Question 1 a):
25 + 87 = **112**

	Hundreds[1]	Tens[1]	Units
		2	5
		8	7
	1	**1**	**2**

Question 1 b):
102 – 79 = **23**

	Hundreds	Tens	Units
	~~1~~	~~1~~0 [9]	12
–		7	9
		2	**3**

Question 1 c):
Calculate 56 × 75 = **4,200**

	Thousands[+1]	Hundreds[4+1]	Tens[3]	Units
			5	6
x			7	5
		2	5	0
+	3	3	2	
	4	**2**	**0**	**0**

Question 1 d):
Calculate 5.6 × 7.5 = **42**

This question is similar to Question 1d, but multiplied by 1/100.

5.6 x 75 = 56 x 1/10 + 75 x 1/ 10
= 56 x 75 x 1/100
= 4,200 x 1/100
= **42**

Question 1 e):
Calculate 216 ÷ 180 = **1.2**

```
          1 . 2
      ┌──────
180   │ 2 1 6
    - │ 1 8 0
      ──────
        3 6 0
      - 3 6 0
        ─────
            0
```

Question 2:
The teacher asked David for a random number from 1 to 5,000 and Venessa another number from 1 to 2,500.
David chose number 4,096 and Venessa chose number is 1,024.
What's the total of their numbers?

= **5,120**

This clearly is an addition question as the question asked for "total". We need to simply lay both numbers neatly in a column format and apply the addition method.

	Hundreds	Tens	Units
	1	1	
	4	0	9	6
+	1	0	2	4
	5	**1**	**2**	**0**

Question 3:
The teacher then asked what's the difference between the number given by David and that of Vanessa's.

= **3,072**

The word "difference" implies the use of "subtraction".

	Hundreds	Tens	Units
4	0	9	6
− 1	0	2	4
3	**0**	**7**	**2**

Question 4:
Jess took £10.00 with her to the local shop. She purchased two packs of crisps costing 74p each, one bottle of milk costing £1.48, three cans of drinks costing 63p each and two packs of ready meal costing £2.38. How much change has she got back?

= **£0.39** or **39p**
(remember to include unit '£' or 'p')

Here we have to work out how much Jess has spent before subtracting this from money she had started with.
2 packs of crisps = 2 x 74p = £1.48
1 bottle of milk = £1.48
3 cans of drinks = 3 x 63p = £1.89
2 packs of ready meals = £2.38 x 2 = £4.76
So in total she had spent

	$Unit^2$	.	$Tenth^3$	Hundredth
	1	.	4	8
	1	.	4	8
	1	.	8	9
+	4	.	7	6
£	**9**	.	**6**	**1**

Now we know she had spent total of £9.61, we can then subtract this amount from £10.00.

	Ten	$Unit^2$	.	$Tenth^3$	Hundredth
	~~1~~	~~0~~ ^{9}	.	~~10~~ ^{9}	10
−		9	.	6	1
£		**0**	.	**3**	**9**