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SelfStudy Series

A3 – **Algebra & Equations**

version 1.04

Contents

PREFACE

Dear parents, guardians and teachers. Thank you for purchasing this study guide directly from algebrains.com. Our SelfStudy guides are available exclusively from algebrains.com (or from our offices) and have been priced to encourage greater accessibility from many students and their families who will benefit from our content. By purchasing directly, you are also contributing and supporting our mission in strengthening the delivery of Maths & Financial Education to children & young-adults in Britain (and throughout the world).

Our SelfStudy series have been written for students as a reference to teach them how to tackle mathematical challenges via step-by-step illustrations. Our materials have been designed to help parents to easily understand the workings too, to help you coach your child.

We have kept the content as concise and as pictorial as possible…so that our examples are easy to follow…therefore easy to understand and apply! We have also decided not to distract the students with elaborate colours as their exam papers will be in black & white.

Should you choose to complement your child's study with our classroom or webinar sessions, your child will also have access to additional illustrated workings for all questions that we shall practise.

Regardless of your child's level, whether a beginner or advanced…we firmly believe that our learning materials coupled with frequent practise will transform your young ones into numerically competent magicians!

Good luck and enjoy learning!

Ying & Jerry

What is Algebra?

In more complex problem-solving situations, where many 'variables' and 'constants' are involved, it is easier to designate letters (usually Greek alphabets) to represent 'variables' and 'constants' to simplify mathematical expressions. Simplifying things makes it easier to solve problems!

In this guide we shall show you how to use algebra to *simplify* problems, and how to *manipulate* algebra to solve problems.

If you join our classroom, webinar or access our Question & Answer materials (visit www.algebrains.com) we shall practise *applying* algebra to derive business and financial solutions. You will find algebra a very powerful too.

Below are some simple depictions of algebra:

(a) **p** + **q** means number p added to second number q

so if p = 5 and q = 3, p + q = 5 + 3 = 8

(b) **p** − **q** means number q subtracted from number p

so if p = 5 and q = 3, p − q = 5 − 3 = 2

(c) **pq** means number p multiplied with number q (no need for 'x' sign)

so if p = 6 and q = 2, p × q = 6 × 2 = 12

(d) $\frac{p}{q}$ means number p divided by number q.

so if p = 6 and q = 2, p ÷q = 6 ÷ 2 = 3

Example: a = apples, p = pears. You have been given the following string:

a + a + p + p + p + a + p + a + p – a – p

We can simplify the above expression by grouping and counting each variable separately. There are actually THREE a's and FOUR p's. Can you see this too?... therefore this expression can be simplified into just 3a + 4p [three apples and four pears].

a + a + p + p + p + a + p + a + p – a – p = **3a + 4p**

The typical way of representing algebra is having the number before letters.

Like and Unlike Terms:

Each 'letter' represent *different* things in an algebraic expression. Unless otherwise stated, letter 'a' does not represent same as letter 'b'. To illustrate, if letter 'a' represent Apples and 'b' represent Bananas… we know Apples are not the same as Bananas, therefore in a mathematical context we should also not mix or confuse letters.

'Like terms' are terms which contain the same letter, therefore, 2a, 5a, 7a are three like terms.

Terms that do not contain the same letters are called 'Unlike terms', thus, 2a, 5b and 7c are three unlike terms.

ONLY LIKE TERMS CAN BE GROUPED (ADDED OR SUBTRACTED FROM EACH OTHER). For example, 5a + 2a = 7a (imagine 5 apples add another 2 apples, we get 7 apples). Conversely, 5a + 2b is NOT the same as 7ab.

Example:

(a) **$5x + 6x$** $= (x + x + x + x + x) + (x + x + x + x + x + x)$

$= (5 + 6)x$

$= 11x$

(b) **7b – 3b** = (7 - 3)b

=4b

(c) **3c – 2c** = 1c

= c **In algebra, 1 x c or 1c is written as just c**

Algebra - Indices:

a x a x a is written as a^3.

The number 3 shows the number of a's that have been MULTIPLED together and is called the INDEX (plural indices). 'a' in this case is called the BASE.

We pronounced a^3 as 'a' to the power of 3.

PLEASE DO NOT CONFUSE a^3 vs 3a – they are NOT the same:

$$a^3 = a \times a \times a \quad \text{versus} \quad 3a = a + a + a$$

Example:

Find the value of b^7 when b = 2

$b^7 = b \times b \times b \times b \times b \times b \times b = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 =$ **128**

Example:

Find the value of $3ab^2$ when a = 7 and b = 2

Steps:

Understand your equation, $3ab^2$ means 3 times a times b to the power of 2.

So that means

$3ab^2 = 3 \times a \times (b \times b) = 3 \times 7 \times (2 \times 2) =$ **84**

Remember in this example, only 'b' is raised to the power of 2, NOT 'a'.

Example:

Find the value of $5m^2n^4$ when m = 2 and n = 3

$5m^2n^4 = 5 \times (2 \times 2) \times (3 \times 3 \times 3 \times 3) = 5 \times 4 \times 81 =$ **1,620**

Algebra - Multiplication:

We have learnt that p x q = pq, but what about when two 'Unlike terms' multiply one another?

Example: 2a x 7b x 3c

Steps:

(1) Study the expression carefully. 2a x 7b x 3c have multiplication throughout. This can be expressed alternatively as

$\Rightarrow$ 2 x a x 7 x b x 3 x c

(2) The numbers (2, 7 and 3) are like terms. Group them together:

$\Rightarrow$ 2 x 7 x 3 x a x b x c

(3) multiply the numbers together, so in this case, 2 x 7 x 3 = 42

$\Rightarrow$ 42 x a x b x c

(4) Remove the 'x' multiplication symbols

$\Rightarrow$ 42abc

$\therefore$ 2a x 7b x 3c = **42abc**

Example: 2a x 3a x 4a

Steps:

(1) The above expression is not interrupted by any addition nor subtraction; only multiplication throughout. Multiply all the numbers together, thus 2 x 3 x 4 = 24

$\Rightarrow$ 24 x a x a x a

(2) a x a x a can be expressed as an indices 'a to the power of 3' or a^3

$\Rightarrow$ $24a^3$

$\therefore$ 2a x 3a x 4a = $\mathbf{24a^3}$

Algebra – Division:

Recall the Division section from our SelfStudy A2-Fractions guide which advises one to consider simplifying fractions as early as possible. We can apply the same methods to algebraic terms expressed in fractions [fractions being another expression of Division].

Example: 16y ÷ 4

Steps:

In algebra we do not use the 'x' multiplication or '÷' division symbols. For the latter, we express divisions in a fractional format:

$$16y \div 4 = \frac{16y}{4} = \frac{4y}{1} = \mathbf{4y}$$

Example: pq ÷ q

Steps:

$$pq \div q = \frac{pq}{q} = \mathbf{P}$$

Expanding Brackets:

Brackets are used to group terms together. You will be asked to expand brackets or simplify expressions by introducing brackets (factoring).

Expanding Brackets:

In order to remove the brackets, we must first appreciate the question

Example: Expand 5(a + b)

Steps

(1) 5(a + b) is the same as 5 x (a + b). This builds on the algebraic convention we have seen earlier where the 'x' multiplication sign is simply omitted.

(2) To expand-out the brackets we multiply x5 against each item inside the bracket

$$5 \times (a + b)$$

$$5(a + b) = \mathbf{5a} + \mathbf{5b}$$

Being aware of how the signs change when you multiply (or divide) values is essential. Below is a quick guide:

$+ \times + = +$	Positive × Positive = Positive
$+ \times - = -$	Positive × Negative = Negative
$- \times + = -$	Negative × Positive = Negative
$- \times - = +$	Negative × Negative = Positive

Example: simplify 4(2a + 8b) + 5(2a - 3b)

4(2a + 8b) + 5(2a - 3b)

$\Rightarrow$ 4 x 2a + 4 x 8b + 5 x 2a + 5 x -3b

$\Rightarrow$ 8a + 32b + 10a – 15b

$\Rightarrow$ (8 + 10)a + (32 - 15)b

$\Rightarrow$ 18a + 17b

$\therefore$ 4(2a + 8b) + 5(2a - 3b) = **18a + 17b**

<u>**Simplify by introducing brackets**</u>:

This is the reverse of expanding brackets…also called Factoring.

Example: simplify 8a + 10b

Steps:

(1) 'a' and 'b' are Unlike terms, but 8 and 10 are both integers and can be factored out.
(2) Both 8 and 10 are multiples of 2, therefore, we can start introducing brackets by pulling 2 outside leaving everything else inside the bracket.

8a + 10b = **2(4a + 5b)**

Try to expand 2(4a + 5b) to see whether you get back to 8a + 10b!

What are Equations?

Equations are algebraic expressions with an EQUAL SIGN in it.

In the next few chapters we are going to *manipulate* algebraic expressions to solve problems using equations.

Rearranging the equation

Sometimes an equation can be "REARRANGED" into a more useful format to deliver the Term you want. For example, this equation shows the relationship between 'F' Fahrenheit and degrees Celsius 'C'.

$$F = 1.8C + 32$$

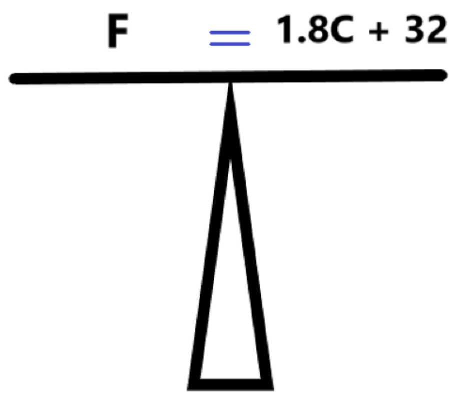
If room temperature is 18°C and you wish to determine its Fahrenheit equivalent, use the above equation 1.8 x 18°C + 32 = 64.4°F

But what if you wish to do the opposite? What is the Celsius of 70°F?

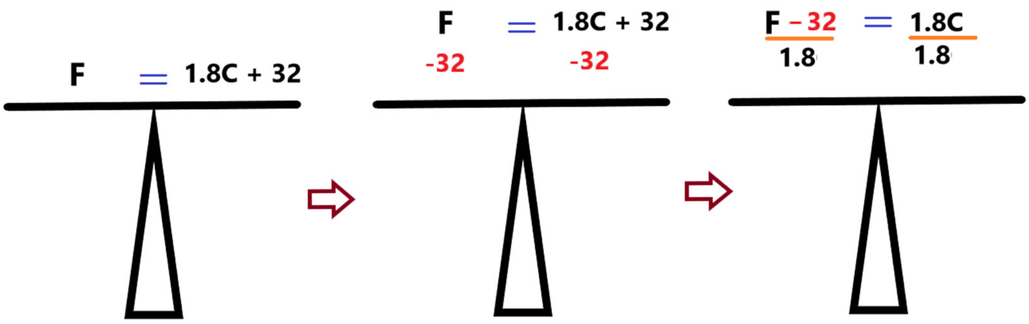
Example: Make C the subject of F = 1.8C + 32

Steps:

(1) It's always easier to imagine the = equal sign as the pivot of a weighing scale, whereby the left & right-hand-sides have equal weight.



(2) The principle is as follows: Whatever we add, subtract, multiply or divide to one side, we must **treat equally to the otherside** of the pivot.



(3) To make C the subject, means we must isolate C exclusively on one-side of the equation.

a. We can remove the numerical 32 from the right. By doing so, we must also remove 32 from the left. This will leave us with $1.8C$ on the right-hand-side.

$$F = 1.8C + 32$$

$$\Rightarrow \quad F - 32 = 1.8C$$

Please observe that subtracting 32 from both sides is the same as moving the +32 from the right-hand-side, dropping this into the left-hand-side AND flipping its sign. If you master this behaviour of manipulation equations…you will master equations very quickly!!

b. Finally, to isolate C we must divide both sides by 1.8

$$F - 32 = 1.8C$$

$$\therefore \quad \frac{F - 32}{1.8} = C$$

To master rearranging equations, I need you to observe the workings illustrated in (3)a and (3)b above. Although I have also shown you pictorial diagrams, such a method is too slow… we need to shift your mind up-a-gear and do this faster by manipulating the equation directly!

Let's look at another example…

Example: Make 'x' the subject from $\boldsymbol{v = \frac{3\,(x+y)}{4}}$

Steps:

(1) Multiply each side by x4 to remove the fraction

$$v = \frac{3\,(x\ + y)}{4}$$

$\Longrightarrow$ $$4v = 3(x\ + y)$$

(2) Divide both side by 3

$\Longrightarrow$ $$\frac{4v}{3} = \ x + y$$

(3) Take 'y' away from both sides

$\therefore$ $$\boldsymbol{\frac{4v}{3} - y = x}$$

Notice that you have been using your algebra to peel away the obstacles between your starting position and getting to your target 'x'. I am hoping you are finding this fascinating and fun!

Summary

Please, please, please practise your algebra (…simplification, bracket expansions, rearranging equations…etc) otherwise this will limit your progression to subsequent topics (SelfStudy A4-Simulateneous Equations). There are many nuances which we have covered in this guide, but you will not become adept at mastering these *unless* you practise.

APPENDIX - formulas

To consolidate your knowledge, you must practise, practise and practise! Enquire about our classroom & webinar courses or Question & Answer materials…visit us at www.algebrains.com

Below we have reproduced the algebraic formulas presented in the SelfStudy A2-Fractions, Ratios & Percentages guide

Addition formula:

$$\frac{a}{b} + \frac{c}{d} = \frac{(a \times d) + (c \times b)}{b \times d}$$

Subtraction formula:

$$\frac{a}{b} - \frac{c}{d} = \frac{(a \times d) - (c \times b)}{b \times d}$$

Multiplication formula:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Division formula:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$$

Changing the display of improper fractions:

$$a\frac{b}{c} = \frac{(a \times c) + b}{c}$$

Practice Questions:

Calculate & show workings…

Question 1:

Calculate (a) $a^2 \times a^3$ when a = 3

(b) $p^2 \times p^5 \div p^5$ when p = 6

Question 2:

Simplify $7(x + 2y) - 2(3x - 2y)$

Question 3:

Calculate the value of 'x' when $3x + 8 = 56$

Question 4:

The equation $v = xyz$ can be used to find the volume of a rectangular box. Rearrange this equation to make 'y' the subject.

Question 5:

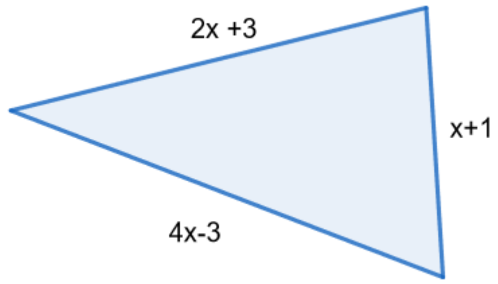
A pair of white shoe laces cost £3, a pair of pink laces costs £5. What is the algebraic expression for the total cost of x pairs of white shoe laces and y pairs of pink shoe laces?

Question 6:

Continuing from Question 5; Helen spent £35 in total on shoe laces. She said she had one more white pair than pink pairs. Can you work out how many pairs of white and pink shoe laces she had purchased?

Question 7:

Produce an algebraic expression for the perimeter of the following triangle



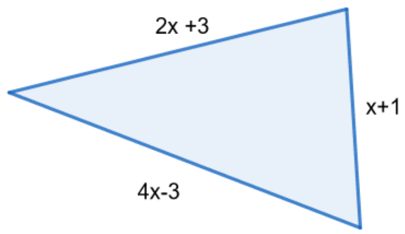
(Not drawn to scale)

Question 8:

If the total perimeter of the triangle is 15 cm. Work out what's x

Answers & Model Workings:

Question 1: Calculate (a) $a^2 \times a^3$ when a = 3 (b) $p^2 \times p^5 \div p^5$ when p = 6	(a) $a^2 \times a^3 = a^{2+3} = a^5$ $= (3 \times 3) \times (3 \times 3 \times 3)$ $= 9 \times 9 \times 3$ $= 81 \times 3$ = **243** (b) $p^2 \times p^5 \div p^5$ $= p^{2+5-5} = p^2$ $= 6^2$ = **36**
Question 2: Simplify $7(x + 2y) - 2(3x - 2y)$	Expand brackets individually and regroup like terms: $7(x + 2y) - 2(3x - 2y)$ $= 7x + 14y - 6x + 4y$ = **13x + 18y**
Question 3: Calculate the value of 'x' when 3x + 8 = 56	$3x + 8 = 56$ $3x = 56 - 8$ $3x = 48$ **x = 16**
Question 4: The equation v = xyz can be used to find the volume of a rectangular box. Rearrange this equation to make 'y' the subject.	We must isolate y on one side of the equation $v = xyz$ $\frac{v}{xz} = y$
Question 5: A pair of white shoe laces cost £3, a pair of pink laces costs £5. What is the algebraic expression for the total cost of x pairs of white shoe laces and y pairs of pink shoe laces?	Denote number of pairs of white shoe lace = x As it costs £3 each, so total cost of white laces would be 3x Denote number of pairs of pink shoe lace = y Total cost of pink shoe laces would be 5x as pink lace costs £5 each. Total costs for x white shoe laces and y pink shoe laces would be **3x + 5y**

Question 6: Continuing from Question 5; Helen spent £35 in total on shoe laces. She said she had one more white pair than pink pairs. Can you work out how many pairs of white and pink shoe laces she had purchased?	We know that white shoe lace is 1 more than pink, [1] $x = y + 1$ And the total cost of £35 is made from 3x and 5y, [2] $3x + 5y = 35$ We have two equations [1] and [2] which can be combined into one: Substitute x from [1] into [2], the equation then becomes: $3(y + 1) + 5y = 35$ [now you only have y to solve] $3y + 3 + 5y = 35$ $8y + 3 = 35$ $8y = 32$ $Y = 4$ Put $y = 4$ into [1], you will get $x = 5$. **So 4 pairs of pink and 5 pairs of white shoe laces.** This are the type of questions you will encounter frequently. Construction of simultaneous equations will be covered in more detail in SelfStudy guide A4, but the concepts behind it draws completely on what you have learnt in this guide.
Question 7: Produce an algebraic expression for the perimeter of the following triangle 	Perimeter = the distance surrounding the shape. We know the length of each sided of the triangle, we just need to add them up. $(2x + 3) + (x + 1) + (4x – 3)$ $= 2x + 3 + x + 1 + 4x –3$ $=$ **$7x + 1$**
Question 8: If the total perimeter of the triangle is 15 cm. Work out what's x	As we know from question 7, total perimeter is $7x + 1$ which also equals to 15. We need to rearrange this to get x. $7x + 1 = 15$ $7x = 15 – 1$ $7x = 14$ **$x = 2$**