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Self Study Series

A4 – Simultaneous equations

version 1.04

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PREFACE

Dear parents, guardians and teachers. Thank you for purchasing this study guide directly from algebrains.com. Our SelfStudy guides are available exclusively from algebrains.com (or from our offices) and have been priced to encourage greater accessibility from many students and their families who will benefit from our content. By purchasing directly, you are also contributing and supporting our mission in strengthening the delivery of Maths & Financial Education to children & young-adults in Britain (and throughout the world).

Our SelfStudy series have been written for students as a reference to teach them how to tackle mathematical challenges via step-by-step illustrations. Our materials have been designed to help parents to easily understand the workings too, to help you coach your child.

We have kept the content as concise and as pictorial as possible…so that our examples are easy to follow…therefore easy to understand and apply! We have also decided not to distract the students with elaborate colours as their exam papers will be in black & white.

Should you choose to complement your child's study with our classroom or webinar sessions, your child will also have access to additional illustrated workings for all questions that we shall practise.

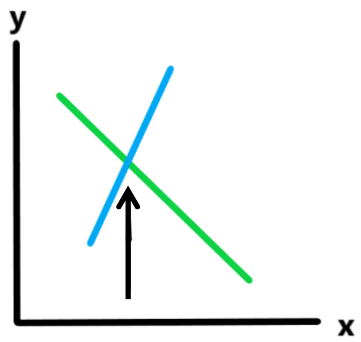
Regardless of your child's level, whether a beginner or advanced…we firmly believe that our learning materials coupled with frequent practise will transform your young ones into numerically competent magicians!

Good luck and enjoy learning!

Ying & Jerry

What are Simultaneous equations?

Simultaneous equations are TWO OR MORE equations which share the same TWO OR MORE variables.



Using the above graph as an illustration: Each line is actually a linear equation. There are two intersecting lines, blue and green lines, thus two equations. Both these equations are influenced by two variables x and y denoted by the axes x and y. The intersecting point is where the value of X and Y as inputs into these equations makes the result of the both equations equal to each other.

Example:

Solve the following simultaneous equations.

$$a + 3b = 17$$

$$2a + 2b = 14$$

Steps:

(1) It's a good habit to always label your equations so you are able to check your own working.

[1] $a + 3b = 17$

[2] $2a + 2b = 14$

Typically there will be 2 unknowns to solve (e.g. a and b)

(2) To commence solving simultaneous equations, we need to ISOLATE one of the unknown variables in an equation. Example as follows:

   a. Looking at both equations, I am going to multiply equation [1] by x2 to make the prefix of 'a' in equation [1] the same as that currently existing in equation [2]. Remember whatever you do to any side of an equation, you must do exactly the same to the other side.
   b. So [1] x 2 = [3]

$\Rightarrow$ [3] $2a + 6b = 34$

Label this as equation [3].

(3) Comparing equation [2] and [3] will be much easier as follows:

[3] $2a + 6b = 34$

[2] $2a + 2b = 14$

The number prefixes in front of the variables have another name in maths….they are called COEFFICIENTS. Since the coefficients of the a's are both the same (therefore both 2a) we can subtract one equation from the other to eliminate the a's!

Multiply each side of equation [2] with -1, then add the result of this equation to [3] to yield equation [4] as below:

[3] $2a + 6b = 34$

- [2] $- 2a - 2b = -14$

[4] $+ 2b = 20$

(4) Equation [4] only has one variable 'b'. Solve for b:

[4] $+ 2b = 20$

$\Rightarrow$ $b = 20/2$

$\therefore$ $b = 5$

(5) If b = 5, substitute this information back into equation [1]

[1] $a + 3b = 17$

$\Rightarrow$ $a + 2x5 = 17$

$\Rightarrow$ $a + 15 = 17$

$\therefore$ $a = 2$

(6) The solution is **a = 2, and b = 5**

Insert these into equations [1] and [2] to check whether they work!

Let's try another example:

Example:

Solve the following simultaneous equations.

[1] $6a - 8b = -20$

[2] $5a + 2b = 18$

Steps:

(1) Choose a term (either 'a' or 'b') and make the coefficient the same. In this example, we shall choose to work on the b's first. Multiply equation [2] by x4 to get the coefficient of 'b' in this equation to 8b (the same magnitude as equation [1])

[2] $5a + 2b = 18$

$\Rightarrow$ [3] $20a + 8b = 72$

(2) Sum equation [1] and [3] to eliminate the b's (no need to subtract as the -8b + 8b will remove the b's to zero)

[1] $6a - 8b = -20$

[3] $20a + 8b = 72$

[4] $26a = 52$

$\Rightarrow$ $a = 52/26$

$\therefore$ $a = 2$

(3) Substitute a = 2 back into say equation [1] or [2] to solve for b

[1] $6a - 8b = -20$

$\Rightarrow$ $12 - 8b = -20$

$\Rightarrow$ $12 + 20 = + 8b$

$\Rightarrow$ $32 = 8b$

$\therefore$ $4 = b$

(4) The solution is **a = 2, and b = 4**

Insert these into equations [1] and [2] to check whether they work!

So how do we use simultaneous equations to solve some everyday questions? Easy, you convert the worded scenario into a mathematical form (the equations), and then solve the equations! …simple!

Example:

Amy was walking in a park and she noticed there was a group of dogs and ducks being gathered together.

She counted that there were a total of 8 heads and 26 legs.

Could you help Amy to work out how many dogs and ducks are there?

Steps:

The wonderful thing about wordy questions is that it allows you to be creative in terms of labelling your unknowns.

(1) Let's use the following to denote our animal friends.

d = dog

k = duck

(2) Since we know there are a total of 8 heads, we can express our first equation as follows:

[1] $d + k = 8$

(3) Secondly, we know dogs have 4 legs and ducks have 2, therefore, we are able to construct our second equation based on this.

[2] $4d + 2k = 26$

Well done! You have converted a worded scenario into a simplified mathematical expression to solve. Solve the following simultaneous equations:

[1] $d + k = 8$
[2] $4d + 2k = 26$

(4) Choose a coefficient of a variable to make the same. In this example we are going to make the coefficient of 'd' the same by multiplying [1] by x4.

	[1]	$d + k = 8$
$\Rightarrow$	[3]	$4d + 4k = 32$

(5) Subtract [2] from [3]

	[3]	$4d + 4k = 32$
	-[2]	$-4d - 2k = -26$
	[4]	$2k = 6$
$\therefore$		$k = 3$

(6) Substitute k = 3 into equation [1] or [2] to derive 'd'.

	[1]	$d + k = 8$
		$d + 3 = 8$
$\therefore$		$d = 5$

(7) The solution is **k = 3, and d = 5. There are total of 5 dogs and 3 ducks.**

Insert these into equations [1] and [2] to check whether they work!

APPENDIX - formulas

To consolidate your knowledge, you must practise, practise and practise! Enquire about our classroom & webinar courses or Question & Answer materials…visit us at www.algebrains.com

Addition formula:

$$\frac{a}{b} + \frac{c}{d} = \frac{(a \times d) + (c \times b)}{b \times d}$$

Subtraction formula:

$$\frac{a}{b} - \frac{c}{d} = \frac{(a \times d) - (c \times b)}{b \times d}$$

Multiplication formula:

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$

Division formula:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{a \times d}{b \times c}$$

Changing the display of improper fractions:

$$a\frac{b}{c} = \frac{(a \times c) + b}{c}$$

Practice Questions:

Calculate & show workings…

Question 1:

Solve the following simultaneous equations

(a) $3x + 5y = 4$
$-4x + 7y = -60$

(b) $8x + 4y = 16$
$12x - 3y = 42$

Question 2:

A machine sells tickets for travel on a tram system. A single ticket costs £2 and a return tickets costs £3.50. In one day, the machines sells 100 tickets and takes £242. How many of each type of ticket were sold?

Question 3:

Find the coordinates of the point of intersection of the lines:

(a) $x + y = 8$ and $y = 2x - 1$
(b) $x + y = 10$ and $y = 2x + 1$

Question 4:

Anna's Aquarium has only two types of creatures: jupiterian jellyfish and ordinary octopus.

Each jupiterian jellyfish has 25 tentacles.

Each ordinary octopus has 8 tentacles.

In Anna's Aquarium, there are 20 creatures and 279 tentacles.

How many jupiterian jellyfish are there in the aquarium?

[NLIGSC]

Answers & Model Workings:

Question 1: Solve the following simultaneous equations (a) $3x + 5y = 4$ $-4x + 7y = -60$ (b) $8x + 4y = 16$ $12x - 3y = 42$	(a) [1] $3x + 5y = 4$ [2] $-4x + 7y = -60$ Since there's + and - on 'x', it will be easier if I make the coefficient of 'x' same by [1]x7 and [2]x5. [3] $12x + 20y = 16$ [4] $-12x + 21y = -180$ $\Rightarrow$ $41y = -164$ $\therefore$ $\mathbf{y = -4}$ Put y into [1] $3x + (5x\text{-}4) = 4$ $3x + (-20) = 4$ $3x = 24$ $\therefore$ $\mathbf{x = 8}$ (b) [1] $8x + 4y = 16$ [2] $12x - 3y = 42$ Again, we have + and - but in y, so I am going to make y the same by [1]x3 and [2]x4. [3] $24x + 12y = 48$ [4] $48x - 12y = 168$ $\Rightarrow$ $72x = 216$ $\therefore$ $\mathbf{x = 3}$ Put x into [1] $24 + 4y = 16$ $\therefore$ $\mathbf{y = -2}$
Question 2: A machine sells tickets for travel on a tram system. A single ticket costs £2 and a return tickets costs £3.50. In one day, the machines sells 100 tickets and takes £242. How many of each type of ticket were sold?	Make s = single ticket r = return ticket [1] $r + s = 100$ [2] $2r + 3.5s = 242$ [1]x2 $\Rightarrow$ [3] $2r + 2s = 200$ [2] – [3] $\Rightarrow$ $1.5s = 42$ $\therefore$ $\mathbf{s = 28}$ Put s into [1] $r + 28 = 100$ $\therefore$ $\mathbf{r = 72}$

	Total of 28 single tickets and 72 return tickets were sold
Question 3: Find the coordinates of the point of intersection of the lines: (a) $x + y = 8$ and $y = 2x - 1$ (b) $x + y = 10$ and $y = 2x + 1$	(a) [1] $x + y = 8$ [2] $y = 2x - 1$ We can simply substitute [2] into [1] to get a new equation with just x as a variable. Then solve for x. $\Rightarrow x + (2x - 1) = 8$ $\Rightarrow 3x = 9$ $\therefore$ $\mathbf{x = 3}$ **and** $\mathbf{y = 5}$ (b) [1] $x + y = 10$ [2] $y = 2x + 1$ Again we substitute [2] into [1] to isolate x. $\Rightarrow x + (2x + 1) = 10$ $\Rightarrow 3x = 9$ $\therefore$ $\mathbf{x = 3}$ **and** $\mathbf{y = 7}$
Question 4: Anna's Aquarium has only two types of creatures: jupiterian jellyfish and ordinary octopus. Each jupiterian jellyfish has 25 tentacles. Each ordinary octopus has 8 tentacles. In Anna's Aquarium, there are 20 creatures and 279 tentacles. How many jupiterian jellyfish are there in the aquarium? [NLIGSC]	Let j = jupiterian jellyfish o = octopus [1] $j + o = 20$ [2] $25j + 8o = 279$ [1]x8 $\Rightarrow$ [3] $8j + 8o = 160$ [2] – [3] $\Rightarrow$ $17j = 119$ $\therefore$ $\mathbf{j = 7}$ Put j into [1] $7 + o = 20$ $\therefore$ $\mathbf{o = 13}$ **There are 7 jellyfish and 13 octopus'**